Gain from Commitment to Different Monetary Policy Targeting Rules
FIRST DRAFT

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Abstract
There has been a large consensus that the objective for monetary policy is to stabilize inflation and the real economy. To maximize social welfare, there is a gain when adopting a commitment rule rather than a discretion policy. However, it is argued that other “discretion” policies such as speed limit, price level, and nominal GDP growth targeting, may be better than the traditional discretion policy and be as good as the traditional commitment policy if not better. Using the New Keynesian model with a purely forward looking Phillips Curve, this paper provides a general analytical solution to assess the difference in welfare of the different policies. It claims that price level targeting would be superior in the sense that the central bank can find the highest social welfare using that policy. It is still the case if social welfare includes volatility of interest rate. However, these alternative policies involve a certain level of commitment. In addition, when trying to reduce volatility of interest rate to avoid the lower bound the central bank may have to change optimal targeting rule.

JEL Codes: E52, E61

Key Words: Monetary Policy, Policy Objectives, Policy Designs, New Keynesian Model

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1 Introduction

The seminal paper written by Kydland and Prescott (1977) is at the origin of a vast literature on the issue of discretion and commitment monetary policies. It analyzed the benefits of implementing a commitment policy as opposed to discretion, and reached some conclusions about the credibility of respective policies. In the paper, discretion is defined as a policy in which this monetary authority change policy depending on the circumstances associated with future events. It offers policy makers more flexibility. At each period, policy makers make a decision that is optimal evaluating the state of the economy at the present time. The agents of the private sector know that the central bank will use the same procedure each period, and the policy makers have no incentive to change policy. The policy is credible.

In contrast, commitment is the promise from policy makers to act consistently according to a certain rule in the future when events arise. It might be counter intuitive to consider that monetary authorities obtain better results in term of welfare by maintaining the same policy over time regardless the event instead of adapting to new situations with all flexibility. The principal argument is that commitment provides a better trade-off between volatility of inflation and volatility of output gap, the two components of social welfare which measures the benefits of respective policies. Discretion conduct of policy leads to stabilization bias which relates to the way the path of the economy takes form to go back to equilibrium. The issue has been described by Svensson (1997), Claırdia et al. (1999), and Woodford (1999). With discretion policy, output gap is brought back to zero relatively quickly, while with a commitment policy it would take longer time. Woodford establishes that commitment corresponds to monetary policy inertia in the sense that output gap is changing more gradually than when using discretion, and it improves welfare. So it would add welfare but it brings then the question of how a central bank can keep the promise to commitment to a specific rule and not be tempted to modify the course of action when it seems beneficial to take advantage of public’s expectations. It is the time inconsistency problem. A central bank could announce of a future policy tightening to curb inflation and by lowering inflation expectation it would ease the trade-off and focus on revitalizing the economy in a more aggressive manner. In other words, the central bank can influence expectation from the private sector in a way that improves welfare and, because of the effect already obtained, the central bank may soon after renege its promise and increase output gap to fight unemployment, and also because there might be political pressure for aggressively decrease unemployment at certain periods. However, the private agents would recognize that central bank will again switch their policy if they promise to fight inflation for another shock. Having lost its credibility, the central bank has no other choice than to aggressively slow the economy to dampen the rise of inflation.

The critical aspect is that a forward looking type discretionary policy is suboptimal when compared with a commitment policy. However, it is not obvious that the central bank should focus on inflation and output gap stabilization. In order to make sure that an optimal policy proves consistent, a central bank with discretionary policy may use an objective function different from the social welfare function (Rogoff (1985), Walsh (1995, 2003), Svensson (1997)). Recent papers have argued that policies such as speed limit, price level, or nominal income growth targeting in a discretion mode may dominate the pure discretion policy.

First, one loss function that a central bank can adopt is one that incorporates the volatility of price level rather than the volatility of inflation. This is an intermediary loss function in the sense that we adopt this policy in a discretion fashion to improve social welfare and bring it as close as possible to the welfare obtained with commitment. Ultimately, the purpose is not to measure the welfare obtained with this intermediary function, but rather to evaluate the original social welfare utilizing the new volatility of inflation and output gap. We can then reconcile optimality and consistency. It is compatible with the idea of selecting a central banker that is more motivated to fight inflation that the social welfare requires so putting more weight on inflation variability. As a consequence, there has been recently a renewed interest in the analysis of price stability rather than inflation stability as an objective in monetary policy. Historically, there has been several monetary regimes that adopted an inflation targeting policy after the end of the Gold Standard, but no institution chose an explicit price level policy except Sweden from 1931 to 1933. However, the idea of embracing a price level targeting regained interest in the monetary policy literature. Particularly, several papers collected in Bank of Canada (1994) have investigated this issue, and Duguay (1994) summarized the debate. The conventional wisdom that emerged from those discussions is the idea
that price level targeting provokes greater variability in inflation and output, or employment. To stabilize price level, it is necessary to have succession of higher than average inflation and lower than average inflation, provoking not only higher variability in inflation but also in output. The advantage of reducing the variability in price level is to have views less distorted by the uncertainty of price changes for long-term nominal contracts and inter-temporal decisions. The potential benefit is to have a greater certainty of the level of prices over time.

Second, Walsh (2002) suggested that choosing the growth in output relative to the growth in potential output rather than the output itself in the loss function may reproduce the same kind of inertia than with the global policy. The central bank would focus on stabilizing not only inflation but also output gap augmented by inflation. Following a cost push shock, inflation would rise and output gap would fall, and under a discretion policy the central bank would stabilize rapidly both variables. However, if policy makers are concerned with stabilizing the change in output gap, then they have more inclined to keep output gap negative a longer time. Mehra (2002) and Paez-Farell (2009) provide some econometric evidence that a speed limit rule does a reasonable job to explain the policy of central banks.

Third, Jensen (2001) considered to add a nominal income growth objective into the loss function of the central bank. The selection of a nominal income growth targeting rule is motivated by its ability to mimic the inertia that appears in the global, price level targeting and speed limit targeting policies. The idea was already mentioned by Bean (1983), and arguments in favor of the policy were developed by Hall and Mankiew (1994) or McCallum and Nelson (1999). There has been a revival of this approach since the issue of lower bound for interest rate. According to McCallum (2011), one of the advantages is to give the central bank a nominal target so it is in monetary terms.

Since the works by Clarida, Gali, and Gertler (1999), Walsh (2003) and Woodford (2003), a lot of studies have advanced the analysis of the optimality of monetary policy by looking at the benefits of different strategies that central banks can adopt, and so using dynamic general stochastic models. It is generally recognized that the improvement depends on the model and the value of the parameters. This paper provides a general analytical solution that allow to comparing the welfare obtained through different monetary policies. It utilizes the same type of formulas developed in Marest and Thurston (2013). Using the basic New Keynesian model, the paper evaluates the welfare gain from the different monetary targeting rules compared to the pure discretion policy, and it argues that the central bank can always find the highest welfare when adopting the price level targeting rule. This is true even if the volatility of interest rate is included in social welfare.

The paper is structures as follows. The basic New Keynesian model is specified in Section 2. Section 3 provides results of the calculation of gain from the global rule against the discretion policy. In Section 4, the performance of the policies considered is analyzed. Section 5 shows the comparison of the different policies when social welfare incorporates volatility of interest rate. Finally, Section 6 concludes.

2 The Basic New Keynesian model

The basic New Keynesian model was introduced by Yun (1996), Rotemberg and Woodford (1997), Goodfriend and King (1997) and others. The version established by Clarida, Gali, and Gertler in “The Science of Monetary Policy” (1999) and further described by Woodford in “Interest and Prices” (2003) is widely used in recent work in policy design. The paper follows the version of the model presented by Walsh (2003). Households provide labor, purchase goods for consumption, hold money and have bonds. Firms produce different goods in a monopolistically competitive goods market, designed by Dixit and Stiglitz (1977). Capital stock is ignored. Price stickiness is specified by the Calvo’s model in which it is assumed that prices adjust infrequently. Each period, there is a constant probability 1-ω that the firm can adjust its price. So there is a probability ω that the firm must keep its price unchanged. Furthermore, it is forward looking because expectations of future variables are in structural equation describing behavior of consumers and firms. We have the traditional log linearized equations:
Equation (4) is the “IS curve”. Equation (5) corresponds to the supply curve; it is the Phillips curve, or inflation curve. Equation (6) is the log linear transformation of equation (2), and determines the quantity of money. It is the demand for money. The following variables, $\varepsilon_t$, $\eta_t$, and $\psi_t$ are white noises.

The other part of the model is the welfare function. Using targeting rules, policy makers try to find the solution of a stochastic dynamic optimal control problem. There is a general consensus among academics (Svensson 1999) to adopt a loss function that depends on the variability of inflation and output gap. By adopting an inflation target rule, central banks have for objective to stabilizing inflation around an inflation target. It is flexible in the sense that it takes also into consideration the stability of the real economy. It is recognized that central banks are concerned also about output gap when implementing their policies. So the loss function takes the form of $L = (\pi_t - \pi^*)^2 + \Gamma x_t^2$ where $\pi^*$ is the inflation target. The inflation target $\pi^*$ here is zero. Woodford (2003) demonstrated that it can be derived by optimizing household utility. In the function loss, $\pi_t$ is the inflation at time $t$, $x_t$ the output gap and $\Gamma$ the relative weight on stabilizing the output gap. $\Gamma$ is the relative importance of output gap volatility in the preference of central bank.

The goal for the central bank is then to maximize the welfare $\max \{ \sum \frac{1}{2} E_t \beta (\Gamma x_t^2 + \pi_t^2) \}$

### 3 Optimal paths and welfare maximizing social welfare

Quantitative results have been put forward by Giannoni (2000), Vestin (2000), Woodford (1999), and Walsh (2003). McCallum and Nelson (2004) found that the difference between discretion and global is most of the time about 15 to 20 percent and could go as low as 2 percent when there is no serial correlation in the ut process. With an auto-regression parameter of $\rho=0.8$, the difference is even bigger. It could have a 3.5 time better result.

Other authors have focused on looking at this difference but with different models, mainly using hybrid Phillips curve and IS curve. In that vein, Dennis (2000) used the model estimated by Rudebusch and found an improvement between 0 and 11% using different policy objectives. Adopting this time models from Fuhrer and Moore (1995), Huh and lansing (2000), and a modified version of CGG, Dennis (2001) found gains ranging from 6 to 9%. Utilizing also a modified version of CGG, Vestin (2001) determined that the gain could range from 0 to 57% depending on the value of the parameters. Also, Ehrmann and Smets (2001) estimated that the improvement is between 17% and 31% with European data. Dennis and Soderstrom (2002) used different models found that the improvement is between 0.28% and 49.6%.

#### 3.1 Paths

**Discretion policy**

First, we study the case of discretionary policy. It is defined as a policy that is implemented at the beginning of each period by policy makers after they examined data from the economy and they optimize their decision. The central bank doesn’t consider future dates, so it cannot affect the expectation of private agents. Individual agents know that the central bank proceeds this way. The expectation from the private sector is that policy makers will continue to adopt that strategy. The policy makers have no incentive to
modify their behavior, and the expectations of the private sector are rational. We should be then able to extract the optimal solution for the relationship between $x_t$ and $\pi_t$ that is established by the central bank. In the discretionary policy case, as it has already been well established that the central bank should minimize $\pi_t^2 + \Gamma x_t^2$ subject to the Phillips curve, or inflation equation. It should thus minimize

$$L = \frac{1}{2} (\pi_t^2 + \Gamma x_t^2) + \psi_t (\pi_t - \kappa x_t - u_t)$$

Here, $\psi_t$ is the Lagrangian multiplier associated with the inflation equation. It should be noticed that $E_t \pi_{t+1}$ disappeared in this equation because the central bank doesn’t consider future dates. The IS curve is not relevant. If the IS curve was included in the constraint, the optimization would reveal that the Lagrange multiplier to that curve would be equal to zero. The first order condition is then

$$x_t = -\frac{\kappa}{\Gamma} \pi_t$$

Following the “Science for Monetary Policy” (Clarida, Gali, and Gertler, 1999), the authors adopt the following pattern: $E_t x_{t+1} = \rho x_t$ and $E_t \pi_{t+1} = \rho \pi_t$. It is a solution of the system of equations. It implies that economic agents expect the central bank to bring back output gap and inflation to their steady state level, and in this form. As a consequence, we can use the IS curve and forward looking Phillips curve to find the optimal paths for inflation, output gap and interest rate. These paths are the optimal paths after a shock $u_t$ from the Phillips curve. This shock $u_t$ represents usually a difference between the marginal cost and the output gap. In this process, deviations of inflation and output gap from steady state in the past are neglected. What counts is to bring back inflation and output back to target. The results can already be found in the “Science for Monetary Policy” (Clarida, Gali, and Gertler, 1999):

$$\pi_t = \frac{\Gamma}{\kappa + \Gamma(1-\beta \rho)} u_t$$

(1)

$$y_t - y_t^f = x_t = -\frac{\kappa}{\kappa + \Gamma(1-\beta \rho)} u_t$$

(2)

$$i_t = \frac{\rho \Gamma (1 + (1-\rho) \frac{\kappa \sigma}{\rho \sigma})}{\kappa^2 + \Gamma(1-\beta \rho)} u_t + \sigma_g$$

(3)

with $u_t = \rho u_{t-1} + \eta_t$ and $\eta_t$ is a white noise process with constant variance $\sigma_u^2$.

We incorporated $\sigma_g$ into the path for interest rate to neutralize the shock from the demand curve. We can find also the real interest rate. We get:

$$r_t = i_t - E_t \pi_{t+1} = (1-\rho) \frac{\kappa \sigma}{\kappa^2 + \Gamma(1-\beta \rho)} u_t + \sigma_g$$

If the economy is subject to an impulse in the Phillips curve, contrary to inflation and output gap, price level is not forced to go back to previous levels, and goes up without limits. We can see that by deducting the path for the price level. We have $p_t = \pi_t + p_{t-1}$ so we obtain:

$$p_t = p_{t-1} + \frac{\Gamma}{\kappa^2 + \Gamma(1-\beta \rho)} u_t$$
In the New Keynesian model, central banks use interest rates as instruments to implement their strategies. However, money is not absent as it is part of the money demand equation. Then we can use the money demand equation to find the optimal path for money:

\[ m_t - p_t = \frac{\sigma}{b} y_t - \frac{1}{b} i_t + \omega_t \]  \hfill (4)

So, using the paths previously shown

\[ p_t = \pi_t + p_{t-1} \]
\[ y_t = x_t + y_{t-1} \]

we get:

\[ m_t = p_t + \frac{\sigma}{b} y_t - \frac{1}{b} i_t \]

from

\[ \frac{v(M_t)}{P_t} = \frac{\sigma}{1 + i_t} \]

\[ \Leftrightarrow m_t = m_{t-1} + \frac{\Gamma - \kappa \sigma - \rho \Gamma (1 + (1 - \rho) \frac{\kappa \sigma}{\rho \Gamma})}{\kappa^2 + \Gamma (1 - \beta \rho)} u_t + \frac{\kappa \sigma + \rho \Gamma (1 + (1 - \rho) \frac{\kappa \sigma}{\rho \Gamma})}{\kappa^2 + \Gamma (1 - \beta \rho)} u_{t-1} \]

\[ + \frac{\sigma}{b} (y_t^f - y_{t-1}^f) - \frac{\sigma}{b} (g_t - g_{t-1}) + \omega_t - \omega_{t-1} \]

**Pre-commitment from the Science of Monetary Policy**

In “The Science of Monetary Policy” (1999), Clarida, Gali and Gertler described a pre-commitment policy in which the target \( x_t \) is contingent on the shock \( u_t \) in this way: \( x_t = \alpha u_t \) and the central bank commit to adopt a path that provides a better result in terms of welfare by being more aggressive in fighting inflation. The paths obtained in this case are:

\[ \pi_t = \frac{\Gamma (1 - \beta \rho)}{\kappa^2 + \Gamma (1 - \beta \rho)^2} u_t \]

And

\[ x_t = -\frac{\kappa}{\kappa^2 + \Gamma (1 - \beta \rho)^2} u_t \]

And we can deduce then the path for interest rate:

\[ i_t = \frac{\kappa \sigma (1 - \rho) + \Gamma (1 - \beta \rho) \rho}{\kappa^2 + \Gamma (1 - \beta \rho)^2} u_t + \sigma g_t \]

We get also the real interest rate:

\[ r_t = i_t - E_t \pi_{t+1} = (1 - \rho) \frac{\kappa \sigma}{\kappa^2 + \Gamma (1 - \beta \rho)^2} u_t + \sigma g_t \]

We have \( p_t = \pi_t + p_{t-1} \) so we obtain:
\[ p_t = \frac{\Gamma(1-\beta \rho)}{\kappa^2 + \Gamma(1-\beta \rho)^2} u_t + p_{t-1} \]

\[ m_t = m_{t-1} + \frac{\Gamma(1-\beta \rho)(1-\frac{\rho}{b}) - \frac{\kappa \sigma}{b} (2-\rho)}{\kappa^2 + \Gamma(1-\beta \rho)^2} u_t + \frac{\kappa \sigma}{b} \frac{(2-\rho) + \Gamma \rho}{\kappa^2 + \Gamma(1-\beta \rho)^2} u_{t-1} \]

\[ + \frac{\sigma}{b} (y_t^f - y_{t-1}^f) - \frac{\sigma}{b} (g_t - g_{t-1}) + \omega_t - \omega_{t-1} \]

We get similar graphs than for the discretion case, with more emphasis on flight of inflation.

**Commitment or global policy:**

Clarida, Gali, and Gertler (1999) also mentioned that the type of commitment described previously is not optimal. A global policy however would consider expectation in the calculation of the first order conditions and would provide an optimal rule under commitment. This time the central bank commit to a path for not only current but also future inflation and output gap. The objective is still to minimize the loss function

\[ E_t(\sum_{i=0}^{\infty} \beta^i (\Gamma x_{t+i}^2 + \pi_{t+i}^2)) \]

The central bank should minimize this loss function subject to the Phillips curve, or inflation equation. The IS curve doesn’t create any constraints on the choice of policy. It should thus minimize

\[ L = E_t(\sum_{i=0}^{\infty} \beta^i (\frac{1}{2} (\pi_{t+i}^2 + \Gamma x_{t+i}^2) + \psi_t (\pi_{t+i} - \beta \pi_{t+i-1} - \kappa x_{t+i} - u_{t+i})) \]

The first order conditions provide the following relationship:

\[ \pi_{t+i} = -\frac{\Gamma}{\kappa} (x_{t+i} - x_{t+i-1}) \quad \text{for all } i \geq 0 \]

Dropping the i to make it easier to read, we can transform this relationship to obtain

\[ x_t = -\frac{\kappa}{\Gamma} \hat{p}_t \quad \text{with } \hat{p}_t = p_t - p_{t-1} \quad \text{and } p_1 \text{ being the price level just before the policy took place.} \]

As a consequence, the central bank is implementing a policy using price level. Originally, the commitment policy time inconsistent in the perspective of Kydland and Prescott (1977). The central bank applies the same rule overtime, for each period including the initial one. If we had to start from an initial period, the condition obtained the first period is different from the following periods. As a consequence, the central bank could re-evaluate the problem in the second period and choose another condition, so in that regard the policy is dynamic inconsistent (McCallum and Nelson 2004). Another way to consider commitment is to omit the initial condition and determine the paths with the other conditions that are supposed to be applied for a long time already. It is considered a “timeless perspective” following Woodford denomination. It is a type of commitment that deals with consistency issue due to initial conditions. The concept is that the policy reflects the kind of commitment in which constraints are the same in all succeeding periods, regardless of the initial condition. It is a common practice to use the timeless approach.

We then have the path for \( \hat{p}_t \) and it is

\[ \hat{p}_t = \delta \hat{p}_{t-1} + \frac{\delta}{1-\beta \delta} u_t \quad \text{with } \delta = \frac{1-\sqrt{1-4\beta a^2}}{2a \beta} \quad \text{and } a = \frac{\Gamma}{\kappa^2 + \Gamma(1+\beta)} \]

We can then deduct the paths for the other variables.
So the price level itself would have the path:

\[ p_t = (1 - \delta) p_{t-1} + \delta \phi_{t-1} + \frac{\delta}{1 - \beta \delta \rho} u_t \]

For the output gap we get:

\[ x_t = \delta x_{t-1} - \frac{\kappa \delta}{(1 - \beta \delta \rho) \Gamma} u_t \]

And for inflation

\[ \pi_t = \delta \pi_{t-1} + \frac{\delta}{1 - \delta \rho} (u_t - u_{t-1}) \]

For the interest rate, we would obtain:

\[ i_t = \delta i_{t-1} + \frac{\delta}{1 - \beta \delta \rho} (\rho + \delta - 1)(1 - \frac{\kappa \sigma}{\Gamma}) u_t - \frac{\delta^2}{1 - \beta \delta \rho} \rho(1 - \frac{\kappa \sigma}{\Gamma}) u_{t-1} + \sigma (g_t - \delta g_{t-1}) \]

We can get also the real interest rate:

\[ r_t = i_t - E_t \pi_{t+1} \]

\[ r_t = \delta r_{t-1} - \frac{\kappa \sigma \delta (\delta + \rho - 1)}{\Gamma (1 - \beta \delta \rho)} u_t + \frac{\sigma \kappa \delta^2}{\Gamma (1 - \beta \delta \rho)} u_{t-1} + \sigma (g_t - \delta g_{t-1}) \]

For the money, we get

\[ m_t = p_t + \frac{\sigma}{b} y_t - \frac{1}{b} i_t \]

\[ m_t = \delta m_{t-1} + (1 - \frac{\kappa \sigma}{\Gamma b} + \frac{1}{b} (1 - \delta - \rho)(1 - \frac{\kappa \sigma}{\Gamma})) \frac{\delta}{1 - \beta \delta \rho} u_t + \frac{\delta^2}{1 - \beta \delta \rho} \frac{\rho}{b}(1 - \frac{\kappa \sigma}{\Gamma}) u_{t-1} \]

\[ + (1 - \delta) p_{t-1} + \frac{\sigma}{b} (y_t^f - \delta y_{t-1}^f) - \frac{\sigma}{b} (g_t - \delta g_{t-1}) + \omega_t - \delta \omega_{t-1} \]

### 3.2 Calibration

The benchmark for calibration is from Rotemberg and Woodford (1997) in which the authors found \( \beta = 0.99, \sigma = 0.16, \rho = 0.35, \kappa = 0.024, \) and \( \Gamma = 0.0048 \). It seems unanimous that \( \beta = 0.99 \). Walsh (2003) select \( \sigma = 1 \), and so the value of \( \sigma \) is considered to be in the range between 0.16 and 1. In the vast majority of the papers, \( \rho \) and \( \lambda \) are in the range of 0.35 and 0.5. \( \kappa \) may vary between 0.001 and 0.1 in general, but we can find some values going to 0.25. McCallum and Nelson (2000) found that a value of \( \kappa \) between 0.01 and 0.05 is consistent with empirical evidence. Roberts (1995) estimated it at 0.075, while Walsh (2002) uses 0.05 and Jensen (2002) chooses 0.1. Also, \( \Gamma \) may be between 0.002 and 0.5. In the lowest part of the ranges of values, Gaspar, Smets, and Vestin use 0.002 while Billy (2002) uses 0.003. Jensen (2002), McCallum and Nelson (2000) use a value as high as 0.25. Bauducco and Caputo use 0.5.

The standard deviation of the white noise is established at 1 but even if it has an influence on the value of welfare itself, it doesn’t change the relative magnitude of the losses.
Figure 1 illustrates the path taken by inflation, output gap, interest rate and real interest rate for the different policies. After a positive cost-push shock, interest rate path in the pre-commitment policy is relatively the same than for discretion, but because of the incorporation of expectation of inflation in the Phillips curve, inflation is lower. Thus real interest rate is higher and output gap in much lower, indicating a more aggressive slowdown of the economy. However, the shapes of the paths are similar. Welfare will be different because of the change in the trade-off between inflation and output gap.

In contrast, when adopting a global policy, the paths are different. Inflation is much lower in the initial phase due to the effect of expectation. The idea is to keep output gap negative for a longer time to fight inflation in a more strongly manner, as we can observe on the graph. It is the inertia described by Woodford. By advertising this commitment, a central bank influences the expectation of agents and inflation is lower than it would have been with a discretion policy. Interest rate is much lower also because policy makers, by guaranteeing commitment in the duration, don’t have to be aggressive in the sense that expectation is doing the work. Further in the process, we can see on the figure that actually we observe deflation, and interest has to be lowered to avoid having a real interest rate too positive that would jeopardize any return of the output gap to zero.

Figure 2: with b=1

Figure 2 shows the main difference of concept of inflation targeting between discretion and global policies. When adopting the global policy, the central bank fights inflation to the point of bringing back price to its
level that it had previous to the shock. The global policy is not explicitly a price level targeting policy, but the result is the same. After an unexpected increase in the price level, a central bank following a global policy will try to restore price level back to where it was before the shock. However, with a discretion policy, there is not such attempt. The central bank only brings back inflation to zero, and as a consequence we observe a drift in the price level.

3.3 Welfare

The issue is then to show how the change in the trade-off between inflation and output gap by adopting different policies modifies welfare. The following formulas have been developed in Marest and Thurston (2013)

**Welfare Discretion**

If inflation and output gap follow those paths, then the resulting loss function is:

\[
L = \pi_t^2 + \Gamma x_t^2
\]

\[
\Leftrightarrow L = \left( \frac{\Gamma}{\kappa^2 + \Gamma(1-\beta\rho)} \right)^2 u_t^2 + \Gamma \left( \frac{\kappa}{\kappa^2 + \Gamma(1-\beta\rho)} \right)^2 u_t^2
\]

\[
\Leftrightarrow L = \frac{\Gamma(\Gamma + \kappa^2)}{(\kappa^2 + \Gamma(1-\beta\rho))^2} u_t^2
\]

In the “Science for Monetary Policy” (Clarida, Gali, and Gertler, 1999), the objective function has the form:

\[
\max -\frac{1}{2} E_t \left( \sum_{i=0}^{\infty} \beta^i (\Gamma x_{t+i}^2 + \pi_{t+i}^2) \right)
\]

Welfare is then:

\[
W_{dis} = -\frac{1}{2} \left( \frac{\Gamma^2}{(\kappa^2 + (1-\beta\rho)^2) \Gamma^2} \right) \left( \frac{1}{1-\beta} \right) \left( \frac{1}{1-\beta} \right) (\kappa^2 + \Gamma) \sigma_\eta^2
\]

**Welfare pre-commitment:**

So we can see that the calculation for the welfare will be similar to the calculation of the welfare for the discretion case, except a few changes.

The factor \( D_d = \frac{\Gamma}{\kappa^2 + \Gamma(1-\beta\rho)} \)

Becomes \( D_{presmp} = \frac{\Gamma}{\kappa^2 + \Gamma(1-\beta\rho)^2} \)

Also we have to multiply the result for inflation by \((1-\beta\rho)\).

Thus, the formula for the welfare of the pre-commitment case according to the paths extracted from the Science of Monetary Policy paper is:

\[
W_{presmp} = -\frac{1}{2} \left( \frac{\Gamma^2}{(\kappa^2 + (1-\beta\rho)^2 \Gamma)^2} \right) \left( \frac{1}{1-\beta\rho^2} \right) \left( \frac{1}{1-\beta} \right) ((1-\beta\rho)^2 + \frac{\kappa^2}{\Gamma}) \sigma_\eta^2
\]
And we can obviously see that this welfare is above the welfare from the discretion policy. It is true if we have:

\[
\frac{(1 - \beta \rho)^2 + \kappa^2}{\Gamma} \frac{\delta}{1 - \beta \rho} + \frac{1}{1 - \beta \rho} \left[ \frac{\delta + \rho - 1}{\Gamma} \kappa^2 \right] \left[ \frac{\delta + \rho - 1}{\Gamma} \kappa^2 \right] < \frac{1 + \kappa^2}{\Gamma} \frac{\delta}{1 - \beta \rho} + \frac{1}{1 - \beta \rho} \left[ \frac{\delta + \rho - 1}{\Gamma} \kappa^2 \right] \left[ \frac{\delta + \rho - 1}{\Gamma} \kappa^2 \right]
\]

Or

\[
0 < \beta \rho \kappa^2 + (1 - \beta \rho)^2 \Gamma \beta \rho \quad \text{which is always true.}
\]

So we have

\[
W_{\text{discretion}} < W_{\text{precommitment}}
\]

**Welfare for global policy:**

The welfare due to inflation is:

\[
W_{\text{global}} = \frac{1}{2} \left( \frac{\delta}{1 - \beta \rho} \right)^2 \left( \frac{1}{\Gamma} \frac{\delta}{1 - \beta} \right) \left[ \frac{\delta + \rho - 1}{\Gamma} \kappa^2 \right] \left[ \frac{\delta + \rho - 1}{\Gamma} \kappa^2 \right] + \frac{2 \delta (\delta + \rho - 1) \rho (\rho - 1) \beta^2}{(1 - \beta \delta^2)(1 - \beta^2 \delta^2 \rho^2)} + \frac{\rho^2 (\rho - 1) \beta^2}{1 - \beta^2 \rho^4} + \frac{2 \delta^2 \rho (\rho - 1) \beta^2}{(1 - \beta \delta^2)(1 - \beta^2 \delta^2 \rho^2)} + \frac{2 \delta \rho^2 (\rho - 1) \beta^2}{(1 - \beta \delta^2)(1 - \beta^2 \delta^2 \rho^2)} \sigma_{\eta}^2
\]

The welfare due to output gap is:

\[
W_{\text{global}} = \frac{1}{2} \left( \frac{\delta}{1 - \beta \rho} \right)^2 \kappa^2 \left( \frac{1}{\Gamma} \frac{\delta}{1 - \beta} \right) \left[ \frac{\delta + \rho - 1}{\Gamma} \kappa^2 \right] \left[ \frac{\delta + \rho - 1}{\Gamma} \kappa^2 \right] + \frac{2 \delta (\delta + \rho - 1) \rho (\rho - 1) \beta^2}{(1 - \beta \delta^2)(1 - \beta^2 \delta^2 \rho^2)} + \frac{\rho^2 (\rho - 1) \beta^2}{1 - \beta^2 \rho^4} + \frac{2 \delta^2 \rho (\rho - 1) \beta^2}{(1 - \beta \delta^2)(1 - \beta^2 \delta^2 \rho^2)} + \frac{2 \delta \rho^2 (\rho - 1) \beta^2}{(1 - \beta \delta^2)(1 - \beta^2 \delta^2 \rho^2)} \sigma_{\eta}^2
\]

So the total welfare is

\[
W_{\text{global}} = W_{\text{global}} + W_{\text{global}}
\]

We can observe with all different values for the parameters that the welfare with the global policy is above the welfare with the pre-commitment from the Science of Monetary Policy and then above the welfare with the discretion policy.

So at the end we obtain

\[
W_{\text{discretion}} < W_{\text{precommitment}} < W_{\text{global}}
\]

The global policy always offers the highest welfare compared to discretion and pre-commitment. We can then see the advantage of price stability or commitment over the other policies. The advantage of using an analytical solution is to be able to test for the robustness of the results. Pre-commitment is always superior to discretion. Here, we have to give values for \(\beta\) and \(\rho\) to evaluate the influence of changes of \(\kappa\) and \(\Gamma\) on welfare. To be conform to the calibration selected above, we choose \(\beta=0.99, \rho=0.5\).
Looking at figure 3, we can confirm the fact that global policy offers a higher welfare than discretion, and it is very robust when changing the values of the variables. There is particularly a zone when $\kappa$ is around 0.03 in which the difference between the global policy and the discretion case is the highest. The more flexible is price changing, or the highest $\kappa$, the higher the welfare. When prices are sticky, it is more difficult for the economy and prices to adjust, and they stay longer away from the paths corresponding to the total flexible case, increasing the loss function and aggravating welfare. The value of the weight $\Gamma$ attributed to volatility of output gap in the policy also plays a role on welfare. The highest is $\Gamma$, the lower is welfare but the change is less dramatic than it is for $\kappa$.

### 3.4 Relationship between $\kappa$ and $\Gamma$

Empirical researches looked for the effect on welfare of different values for $\kappa$ and $\Gamma$. However, Woodford demonstrated that there is a relationship between these two variables.

We have the expression

$$
\kappa = \frac{(1-\omega)(1-\omega\beta)}{\omega} \frac{\sigma + \eta}{1 + \eta \theta} 
$$

$$
\Gamma = \frac{(1-\omega)(1-\omega\beta)}{\omega} \frac{\sigma + \eta}{(1 + \eta \theta) \theta} 
$$

Or

$$
\Gamma = \frac{\kappa}{\theta} 
$$

Using the calibration of $\theta=7.88$, we can observe the influence of $\Gamma$ or $\kappa$ assuming that relationship is correct. On the left of figure 4, we have the welfare for the different policies depending on $\kappa$ only. On the right, we have the difference in percentage between the discretion and global policies.
3.5 Robustness

The other parameters such as $\rho$, $\beta$, or $\theta$ don’t play a major role in how to select the different policies.

The figures show that the highest the persistence, the lowest welfare is. Again, similar to the reason behind the influence of price stickiness, inflation and output gap adjust much more slowly with a higher persistence. As a consequence, welfare is lower because the difference between the evolution of the economy and inflation and the path when prices are flexible is larger for a longer period of time. Furthermore, the difference between global and discretion is accentuated when persistence is higher. The figures indicate also that the higher $\beta$ is, the lower welfare is. Indeed, welfare is negative of the discounted value of the future volatilities of inflation and output gap. As a consequence, the higher the discount factor, the lower the welfare. It corresponds to the situation in which the society in general is discounting less the future losses, putting more value on the losses.
Influence of $\theta$:

Using the relationship between $\kappa$ and $\Gamma$ according to Woodford, there we can observe the influence of $\theta$ on welfare. It seems that it doesn’t have any strong influence. However, $\kappa$ has also $\theta$ in its term, so it may be better if we use a figure that shows how welfare changes when $\theta$ and $\omega$ change.

Figure 6:

3.6 Trade-off between volatilities of inflation and output gap.

The gain in welfare from the commitment policy has been attributed to a better trade-off between the volatility of inflation and volatility of output gap. The volatility of inflation is decreasing and the volatility of output gap is increasing but in a more favorable way for the calculation of welfare.

Volatility of Inflation

We can calculate the volatility of inflation with the same methodology than for the calculation of welfare. (see appendix).

For discretion, we get:

$$E_t(\pi_{t+n}^2) = \frac{D_d^2}{1-\rho^2}\sigma_y^2 \text{ with } D_d = \frac{\Gamma}{\kappa^2 + \Gamma(1-\beta\rho)}$$

For the pre-commitment case, we just have to transform $D_d$ as:

$$D_{pre} = \frac{\Gamma(1-\beta\rho)}{\kappa^2 + \Gamma(1-\beta\rho)^2}$$

So we would get:

$$E_t(\pi_{t+n}^2) = \frac{D_{pre}^2}{1-\rho^2}\sigma_y^2 \text{ with } D_{pre} = \frac{\Gamma(1-\beta\rho)}{\kappa^2 + \Gamma(1-\beta\rho)^2}$$
For the global case, we get:

\[
D_g = \frac{\delta}{1 - \beta \delta \rho}
\]

\[
E_t(\pi_{t+n}^2) = D_g \left( 1 + \frac{(\delta + \rho - 1)^2}{1 - \delta^2} + \frac{2\delta(\delta + \rho - 1)\rho(\rho - 1)}{(1 - \delta^2)(1 - \delta \rho)} + \rho^2(\rho - 1)^2 + \frac{(\delta + \rho)^2 \rho^2(\rho - 1)^2}{1 - \delta^2} \right)
\]

\[+ \frac{2\delta^5(\rho - 1)^2}{(1 - \delta^2)(1 - \delta \rho)(1 - \rho^2)} + \frac{\rho^6(1 - \rho)}{(1 - \delta^2)(1 + \rho)} + \frac{2\delta^2 \rho^4(1 - \rho)^2}{(1 - \delta^2)(1 - \delta \rho)} \sigma_\eta^2
\]

**Volatility of output gap**

In contrast, volatility of output gap should decrease when adopting commitment policy. Svensson argued that welfare could improve with commitment having both volatility of inflation and volatility of output gap increase. It is what he could a “free lunch”. However, he obtained this result with a backward looking Phillips curve.

In the discretion case, we have also

\[
x_t = -\frac{\kappa}{\Gamma} \pi_t
\]

So

\[
x_t = -\frac{\kappa}{\kappa^2 + \Gamma(1 - \beta \rho)} u_t
\]

So we can use the results we had for inflation. We just have to change \( D_d \) by:

\[
D_s = -\frac{\kappa}{\Gamma} D_d
\]

\[
E_t(x_{t+n}^2) = \left( \frac{\kappa^2}{\Gamma^2} \right) \frac{D_d^2}{1 - \rho^2} \sigma_\eta^2 \text{ with } D_d = \frac{\Gamma}{\kappa^2 + \Gamma(1 - \beta \rho)}
\]

For the pre-commitment case, we have a similar situation in the sense that we get the volatility of \( x_t \) using the volatility of \( \pi_t \) but we just have to transform \( D_{pre} \) by:

\[
D_{pre} = -\frac{\kappa}{\kappa^2 + \Gamma(1 - \beta \rho)^2}
\]

we get

\[
E_t(x_{t+n}^2) = \frac{D_{pre}^2}{1 - \rho^2} \sigma_\eta^2 \text{ with } D_{pre} = -\frac{\kappa}{\kappa^2 + \Gamma(1 - \beta \rho)^2}
\]
For the global case, 

with 

\[ D_{gx} = -\frac{\kappa}{\Gamma} D_g \]

With 

\[ D_g = \frac{\delta}{1 - \beta \delta \rho} \]

We get 

\[ E_r(x_{t+n}^2) = D_g \left( 1 + \frac{(\delta + \rho)^2}{1 - \delta^2} + \frac{2\delta(\delta + \rho)\rho^2}{(1 - \delta^2)(1 - \delta \rho)} + \rho^4 + \frac{(\delta + \rho)^2 \rho^4}{1 - \delta^2} \right) \]

\[ + \frac{2\delta \rho^7}{(1 - \delta^2)(1 - \delta \rho)(1 - \rho^2)} + \frac{\rho^8}{(1 - \delta^2)(1 - \rho^2)} + \frac{2\delta^2 \rho^6}{(1 - \delta^2)(1 - \delta \rho)} \sigma_{\eta}^2 \]

Figure 7: The chart provides us a perspective of the difference between the policies.

Figure 7 confirms that the volatility of inflation is decreasing when adopting commitment policy. It is particularly true for lower \( \kappa \) and \( \Gamma \). It is noticeable however, that when prices are more flexible volatility of inflation is higher for global than for pre-commitment.
Volatility of interest rate with shock \( g_t \):

If we want to incorporate the IS shock, then we just add \( \sigma g_t \) to it for the discretion, pre-commitment, and global cases.

For the discretion case, we then get:

\[
i_t = \frac{\rho \Gamma (1 + (1 - \rho) \frac{\kappa \sigma}{\rho \Gamma})}{\kappa^2 + \Gamma (1 - \beta \rho)} u_t + \sigma g_t, \text{ with } u_t = \rho u_{t-1} + \eta_t \text{ and } g_t = \lambda g_{t-1} + \varepsilon_t,
\]

so the total variance is:
\[ E_t(i_{t+n}^2) = \frac{D_i^2}{1-\rho^2} \sigma^2 + \frac{\sigma^2}{1-\lambda^2} \sigma^2 \] with \[ D_i = \frac{\rho \Gamma(1+(1-\rho)\frac{\kappa \sigma}{\rho})}{\kappa^2 + \Gamma(1-\beta \rho)} \]

For the pre-commitment case, we proceed the same way and we get:

\[ E_t(i_{t+n}^2) = \frac{D_{pre}^2}{1-\rho^2} \sigma^2 + \frac{\sigma^2}{1-\lambda^2} \sigma^2 \]

For the global case, we have:

\[ i_t = \delta \left( \frac{(\rho + \delta - 1)(1-\frac{\kappa \sigma}{\Gamma})u_t - \frac{\delta^2}{1-\beta \rho} + \sigma(g_t - \delta g_{t-1})}{1-\delta^2} \right) \]

So at each step, the \( g_{t-1} \) due to \( i_{t-1} \) disappears because of \( \delta g_{t-1} \) at the end of the expression and we are left with \( \sigma g \) as with the other cases, so the variance is:

\[ E_t(i_{t+n}^2) = (D_{gt}^2 + \frac{Y^2}{1-\delta^2} + \frac{2\delta Z}{(1-\delta^2)(1-\delta \rho)} + \frac{Z^2}{1-\delta^2}) \]

\[ + \frac{2\delta \rho^2 Z^2}{(1-\delta^2)(1-\delta \rho)(1-\rho^2)} + \frac{\rho^4 Z^2}{(1-\delta^2)(1-\rho^2)} + \frac{2\delta^2 \rho^2 Z^2}{(1-\delta^2)(1-\delta \rho)(1-\delta \rho)} \sigma^2 \]

\[ + \frac{\sigma^2}{1-\lambda^2} \sigma^2 \]

Volatility of real interest rate without gt:

In the discretion case, with:

\[ D_r = \frac{(1-\rho)\kappa \sigma}{\kappa^2 + \Gamma(1-\beta \rho)} \]

we get

\[ E_t(r_{t+n}^2) = \frac{D_r^2}{1-\rho^2} \sigma^2 \] with \[ D_r = \frac{(1-\rho)\kappa \sigma}{\kappa^2 + \Gamma(1-\beta \rho)} \]

For the pre-commitment case, we just have to transform \( D_r \) as:

\[ D_{pre} = \frac{\kappa \sigma(1-\rho)}{\kappa^2 + \Gamma(1-\beta \rho)^2} \]

So we would get:

\[ E_t(r_{t+n}^2) = \frac{D_{pre}^2}{1-\rho^2} \sigma^2 \] with \[ D_{pre} = \frac{\kappa \sigma(1-\rho)}{\kappa^2 + \Gamma(1-\beta \rho)^2} \]

For the global case,
with
\[ D_{gr1} = -\frac{k\sigma\delta}{\Gamma(1-\delta\beta)} (\rho + \delta - 1) \]
\[ Y = \delta D_{gr1} + \rho D_{gr1} - D_{gr2} \quad \text{with} \quad D_{gr2} = -\frac{k\sigma\delta^2}{\Gamma(1-\beta\delta)} \rho \]
\[ Z = \rho (\rho D_{gr1} - D_{gr2}) \]
We get
\[ E_t (r_{t+n}^2) = (D_{gr1}^2 + \frac{Y^2}{1-\delta^2} + \frac{2\delta Y Z}{(1-\delta^2)(1-\delta\phi)} + Z^2 + \frac{(\delta + \rho)^2 Z^2}{1-\delta^2} + \frac{2\delta^2 \rho^2 Z^2}{(1-\delta^2)(1-\delta\phi)}) \sigma^2 \]

If we want to incorporate the IS shock, then we just add \( \sigma_g \) to it for the discretion, pre-commitment, and global cases.

As for the volatility of interest rate, we just have to add \( \frac{\sigma^2}{1-\lambda^2} \sigma^2 \) to the volatilities calculated above.

Figure 10:

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{percentage_gain}
\caption{In percentage gain: With Woodford relationship}
\end{figure}

4 Gain from alternative policies

4.1 Speed limit

As mentioned by Woodford (1999) and McCallum (1999), the optimal results are obtained with monetary policies that are history-dependent. Walsh (2002) and also McCallum (2004) proposed a speed limit targeting rule implemented in a discretion mode to provide the kind of inertia that would improve social welfare. The central bank reacts to the change of output gap rather than its level. Using a hybrid Phillips curve, they conclude that this rule follows closely the global policy. Stracca (2006) reaches also the same conclusion using data from the euro area. It was also confirmed by Yetman (2006). Using neo-classical model, Hatcher (2007) reaches similar conclusion. In this paper, using a forward Phillips curve, it argues that the central bank can adopt a weight on output that makes that policy superior to the pure discretion policy. However, it cannot do better than the global policy.
We should be then able to extract the optimal solution for the relationship between $x_t$ and $\pi_t$ that is established by the central bank. In the discretionary policy case, as it has already been well established that the central bank should minimize $\pi_t^2 + \Gamma xx_t^2 + \Gamma it^2$ subject to not only the Phillips curve but also the IS curve because it involves interest rate. It should thus minimize

$$L = E \sum_{j=0}^{\infty} \beta^j \left[ -\frac{1}{2} (\pi_{t+j}^2 + \Gamma (x_t - x_{t-1})^2) + \psi_{t+j} (\pi_{t+j} - \beta \pi_{t+j+1} - \kappa x_{t+j} - u_{t+j}) \right]$$

For the discretion case, we get the following first order conditions:

By deriving by $\pi_{t+j}$:

$$-\pi_{t+j} + \psi_{t+j} = 0$$

By deriving by $x_{t+j}$:

$$-\Gamma (x_{t+j} - x_{t+j-1}) - \kappa \psi_{t+j} = 0$$

So we get the following first order condition:

$$\pi_t = -\frac{\Gamma}{\kappa} (x_t - x_{t-1})$$

The loss function takes the form:

$$L = \pi_t^2 + \Gamma s \left( x_t - x_{t-1} \right)$$

The first order condition under a discretion policy is the same as the one from the global policy.

$$\pi_t = -\frac{\Gamma}{\kappa} (x_t - x_{t-1})$$

The paths are then identical to the ones from the global policy. Incorporating the paths of inflation and output gap with a weight $\Gamma_{sl}$ into the social welfare that has a weight of $\Gamma_{sl} = \frac{\kappa}{\theta}$, the welfare from the speed limit policy is always superior to the welfare of the pure discretion policy when $\Gamma_{sl}$ is low enough, but it is always lower than the welfare from the global policy (See figure). The inertia created by using the change of output gap instead of its level improve welfare of the discretion policy but the relationship Woodford (2003) found between $\Gamma$ and $\kappa$ provides the optimal weight on output.

Figure 11: Salmon is speed limit

In percentage gain:
4.2 Price level targeting

Some authors (Fisher (1994), Haldane and Salmon (1995) and Kiley (1998)) use the idea of an increase of volatility of the output gap to oppose a price level targeting regime. In contrast, Dittmar, Gavin and Kydland (1999) and Svensson (1999) argue that this policy shows a reduction of the volatility of not only inflation but also output gap. However, those results are obtained using a neo-classical Phillips curve. On the other hand, Dittmar and Gavin (2000) and Vestin (2003) found that this result holds with the New Keynesian Phillips curve, which is forward looking. The policy provides a more favorable combination of volatility of inflation and output gap.

The stickiness of prices may be a factor that plays a role in the decision to target price level. According to the New Keynesian Phillips curve, it is the difference between current inflation and expected inflation that makes output gap fluctuating (Kiley 1998). When future inflation is expected to increase, then output gap decreases. Due to the fact that price are sticky, firms increase their price to response to anticipated increase of future demand. As a consequence, the aggregate supply shifts negatively, and it as for effect to decrease current output. Price level targeting implies deflation after an initial inflationary shock, so it provokes more volatility in output gap. Svensson (1996) weighted on the debate by claiming that policy makers may get a “Free Lunch” by adopting a price level targeting. However, he was using “Neoclassical” Phillips curve with incorporates lag inflation.

First, we study the case of discretionary policy. It is defined as a policy that is implemented at the beginning of each period by policy makers after they examined data from the economy and they optimize their decision. The central bank doesn’t consider future dates, so it cannot affect the expectation of private agents. Individual agents know that the central bank proceeds this way. The expectation from the private sector is that policy makers will continue to adopt that strategy. The policy makers have no incentive to modify their behavior, and the expectations of the private sector are rational.

We should be then able to extract the optimal solution for the relationship between $x_t$ and $\pi_t$ that is established by the central bank.

We should be then able to extract the optimal solution for the relationship between $x_t$ and $\pi_t$ that is established by the central bank. In the discretionary policy case, as it has already been well established that the central bank should minimize $p_t^2 + \Gamma x_t^2$ subject to the Phillips curve, or inflation equation. It should thus minimize

$$L = \frac{1}{2} (p_t^2 + \Gamma_p x_t^2) + \psi_t (p_t - p_{t-1} + \beta \rho_t - \kappa x_t - u_t)$$

The first order condition:

$$p_t = \frac{(1 + \beta)\Gamma_p}{\kappa} x_t$$

So we obtain

$$p_t = \hat{\rho} p_{t-1} + \frac{\delta}{1 - \beta \hat{\rho}} u_t$$

with $\delta = \frac{1 - \sqrt{1 - 4 \beta a^2}}{2a \beta}$ and $a = \frac{(1 + \beta)\Gamma_p}{\kappa^2 + \Gamma(1 + \beta)^2}$

As a consequence:

$$x_t = \hat{\delta} x_{t-1} = \frac{\kappa \delta}{\Gamma_p (1 - \beta \hat{\rho})(1 + \beta)} u_t$$

$$\pi_t = \hat{\delta} \pi_{t-1} + \frac{\delta}{1 - \beta \hat{\rho}} (u_t - u_{t-1})$$

$$i_t = \hat{\delta} i_{t-1} + \frac{\delta}{1 - \beta \hat{\rho}} (1 - \frac{\kappa \sigma}{\Gamma_p (1 + \beta)}) (\delta - 1 + \rho) u_t - \frac{\delta^2 \rho}{1 - \beta \hat{\rho}} (1 - \frac{\kappa \sigma}{\Gamma_p (1 + \beta)}) u_{t-1} + \sigma (g_t - \delta g_{t-1})$$
We can find also the path for the real interest rate and money. We get:

\[
    r_t = \delta r_{t-1} - \frac{\delta}{1 - \beta \delta} \frac{\kappa \sigma}{\Gamma_p (1 + \beta)} (\delta - 1 + \rho) u_t + \frac{\delta^2 \rho}{1 - \beta \delta} \frac{\kappa \sigma}{\Gamma_p (1 + \beta)} u_{t-1} + \sigma (g_t - \delta g_{t-1})
\]

\[
    m_t = \delta m_{t-1} + (1 - \frac{\kappa \sigma}{b \Gamma_p (1 + \beta)}) + \frac{1}{b} (1 - \delta - \rho) (1 - \frac{\kappa \sigma}{\Gamma_p (1 + \beta)}) \frac{\delta}{1 - \beta \delta} u_t + \frac{\delta^2 \rho}{b (1 - \beta \delta)} (1 - \frac{\kappa \sigma}{\Gamma_p (1 + \beta)}) u_{t-1}
    + (1 - \delta) p_{t-1} + \frac{\sigma}{b} (y_{f,t} - \delta y_{f,t-1}) - \frac{\sigma}{b} (g_t - \delta g_{t-1}) + \omega_t - \delta \omega_{t-1}
\]

The paths shown in Figure 12 have the same characteristics than the ones from the global policy. Inertia seems even more pronounced that with the global or speed limit policies.

Figure 12:

In Figure 13, welfare with price level targeting is in blue and it has the same kind of pattern than the welfare from other policies. However, price level targeting seems to be a superior policy. For any price stickiness \( \kappa \), the central bank can select a low weight \( \Gamma_p \) on output gap that would provide a welfare that is higher than welfare with not only discretion but also global policies. Blake (2001) demonstrated that the global first order condition found when maximizing social welfare is not the optimal solution. It is possible that the targeting of other welfare function could provide better outcome.
Figure 13: Blue is price level targeting

Figure 14 shows the volatilities of the different policies. The volatility of inflation for the price level targeting is higher than for the global policy. However, the volatility of output gap is lower, and the trade-off gives a better welfare.

Figure 14:

For output gap

4.3 Nominal GDP growth

Stability of the nominal income targeting regime has been studied for a while. However, the result depends mainly on the model adopted, and particularly the form of the Phillips curve. Ball (1999) used a backward looking Phillips curve and concluded that not only the nominal income targeting policy is not efficient but also it creates instability in the sense that the variances of inflation and output gap are infinite. In contrast, McCallum (1997) argue that stability depends on the specifications of the Phillips curve. With a forward looking model, he finds that nominal income targeting is not instable. Dennis (2001) reaches similar conclusion. Malik (2005) found that nominal income targeting performs better than the price level targeting rule, using a continuous New Keynesian model. Incorporating endogenous persistence, Hanson and
Kapinos (2006) showed that the alternative rules such as nominal income growth targeting do not perform as well. If there is a forward looking component in the model, it is enough to make the system stable. It depends then on the form of the Phillips curve.

Loss function:
\[ L = \pi_t^2 + \Gamma_{gdp}(\pi_t + x_t + x_{t-1}) \]

First order condition:
\[ \pi_t = -\frac{\Gamma_{gdp}(1+\kappa)}{\kappa + \Gamma_{gdp}(1+\kappa)}(x_t - x_{t-1}) \]

For the discretion case, we get the following first order conditions:

By deriving by \(\pi_{t+j}\):
\[ -\pi_{t+j} - \Gamma(\pi_{t+j} + x_{t+j} - x_{t+j-1}) + \psi_{t+j} = 0 \]

By deriving by \(x_{t+j}\):
\[ -\Gamma(\pi_{t+j} + x_{t+j} - x_{t+j-1}) - \kappa\psi_{t+j} = 0 \]

So we get the following first order condition:
\[ \pi_t = -\frac{\Gamma(1+\kappa)}{\kappa + \Gamma(1+\kappa)}(x_t - x_{t-1}) \]

We can transform this relationship to obtain
\[ x_t = -\frac{\kappa + \Gamma(1+\kappa)}{\Gamma(1+\kappa)}\hat{p}_t \]
with \(\hat{p}_t = p_t - p_{t-1}\) and \(p_{t-1}\) being the price level just before the policy took place.

As a consequence, the central bank is implementing a policy using price level.

We then have the path for \(\hat{p}_t\) and it is
\[ \hat{p}_t = \delta p_{t-1} + \frac{\delta}{1 - \beta\delta\rho} u_t \]
with \(\delta = \frac{1 - \sqrt{1 - 4\beta\sigma^2}}{2a\beta}\) and \(a = \frac{\Gamma(1+\kappa)}{\kappa^2 + \Gamma(1+\kappa)(1+\beta + \kappa)}\)

We can then deduce the paths for the other variables.

For the output gap we get:
\[ \chi_t = \delta\chi_{t-1} - \frac{\kappa\delta}{(1 - \beta\delta\rho)\Gamma} u_t \]

And for inflation
\[ \pi_t = \delta\pi_{t-1} + \frac{\delta}{1 - \delta\phi\rho}(u_t - u_{t-1}) \]

or \(\pi_t = -(1 - \delta)\hat{p}_{t-1} + \frac{\delta}{1 - \beta\delta\rho} u_t\)

For the interest rate, we would obtain:
\[ i_t = -(1 - \delta)(1 - \frac{\kappa\sigma}{\Gamma})\hat{p}_t + \frac{\delta}{1 - \beta\delta\rho} \rho(1 - \frac{\kappa\sigma}{\Gamma})u_t + \sigma g_t \]

Or
\[ i_t = \delta\rho_t - \frac{\delta}{1 - \beta\delta\rho} (\rho + \delta - 1)(1 - \frac{\kappa\sigma}{\Gamma})u_t - \frac{\delta^2}{1 - \beta\delta\rho} \rho(1 - \frac{\kappa\sigma}{\Gamma})u_{t-1} + \sigma(g_t - \delta g_{t-1}) \]

The paths are represented in the following graph:

We can get also the real interest rate:
\[ r_t = i_t - E_t\pi_{t+1} \]
\[ r_t = \delta r_{t-1} - \frac{\kappa\sigma\delta(\delta + \rho - 1)}{\Gamma(1 - \beta\delta\rho)} u_t + \frac{\kappa\sigma\delta^2\rho}{\Gamma(1 - \beta\delta\rho)} u_{t-1} + \sigma(g_t - \delta g_{t-1}) \]

For the money, we get
\[ m_t = p_t + \frac{\sigma}{b} y_t - \frac{1}{b} \bar{y}, \]

and we get

\[ m_t = \delta m_{t-1} + \left(1 - \frac{\kappa \sigma}{Tb} + \frac{1}{b} (1 - \delta - \rho)(1 - \frac{\kappa \sigma}{T}) \right) \frac{\delta}{1 - \beta \delta \rho} u_t + \frac{\delta^2}{1 - \beta \delta \rho} \frac{\rho}{b} (1 - \frac{\kappa \sigma}{T}) u_{t-1} \]

\[ + \left(1 - \delta \right) p_{t-1} + \frac{\sigma}{b} (y_t - \bar{y}_{t-1}) - \frac{\sigma}{b} \left( g_t - \bar{g}_{t-1} \right) + \omega_t - \delta \omega_{t-1} \]

The paths for nominal income targeting offer also the kind of inertia that is similar to global, price level targeting and speed limit policies.

**Figure 15:**

![Path Nominal GDP Growth](image1)

**Figure 16:** Aquamarine is nominal growth targeting

![Aquamarine](image2)

Figure 16 shows that generally nominal income targeting is superior to the pure discretion policy, particularly when price stickiness \( \kappa \) is lower and the weight on output is larger.

**Figure 16:** Aquamarine is nominal growth targeting
4.4 Welfare gain with different policies

Figure 17 shows how price level targeting is a superior policy. The central bank can choose a low enough weight $\Gamma_p$ on output to get a higher welfare.

Figure 17:

5 Social welfare with volatility of interest rate:

In this loss function, it is assumed that the central bank is also concerned by interest rate volatility. Initially, Woodford and Rotemberg (1998) raised the concern that interest rate could hit the zero lower bound and suggested that incorporating the interest rate volatility into welfare would minimize that risk. Woodford (1999) also argued that the addition of an interest rate smoothing objective in the loss function would introduce inertia and improve welfare.

By adopting an inflation target rule, central banks have for objective to stabilizing inflation around an inflation target. It is flexible in the sense that it takes also in consideration the stability of the real economy. It is recognized that central banks are concerned also about output gap when implementing their policies. Using targeting rules, policy makers try to find the solution of a stochastic dynamic optimal control problem. There is a general consensus among academics to adopt a loss function that depends on the variability of inflation and output gap. So the loss function takes the form $L = (\pi_t - \pi^*)^2 + \Gamma x_t^2$ where $\pi^*$ is the inflation target. The inflation target $\pi^*$ here is zero.

In the function loss, $\pi_t$ is the inflation at time $t$, $x_t$ the output gap and $\Gamma$ the relative weight on stabilizing the output gap.

The goal for the central bank is then to maximize the welfare

$$\max -\frac{1}{2} \mathbb{E}_t \left( \sum_{i=0}^{\infty} \beta^i \left( \pi_{t+i}^2 + \Gamma_x x_{t+i}^2 + \Gamma_{i} i_{t+j}^2 \right) \right)$$

5.1 Paths and Welfare

Discretion policy

First, we study the case of discretionary policy. It is defined as a policy that is implemented at the beginning of each period by policy makers after they examined data from the economy and they optimize their decision. The central bank doesn’t consider future dates, so it cannot affect the expectation of private agents. Individual agents know that the central bank proceeds this way. The expectation from the private sector is that policy makers will continue to adopt that strategy. The policy makers have no incentive to modify their behavior, and the expectations of the private sector are rational.
We should then be able to extract the optimal solution for the relationship between $x_t$ and $\pi_t$ that is established by the central bank. In the discretionary policy case, as it has already been well established that the central bank should minimize $\pi_t^2 + \Gamma_x x_t^2 + \Gamma_i i_t^2$ subject to not only the Phillips curve but also the IS curve because it involves interest rate. It should thus minimize

$$L = \sum_{j=0}^{+\infty} \beta^j \left[ -\frac{1}{2} (\pi_{t+j}^2 + \Gamma_x x_{t+j}^2 + \Gamma_i i_{t+j}^2) + \nu_{t+j} (x_{t+j} - x_{t+j+1} + \frac{1}{\sigma} (i_{t+j} - \pi_{t+j+1}) - g_{t+j}) ight]$$

$$+ \psi_{t+j} (\pi_{t+j} - \beta \pi_{t+j+1} - \kappa \nu_{t+j} - u_{t+j})]$$

For the discretion case, we get the following first order conditions:

By deriving by $i_{t+j}$: 
$$-\Gamma_i i_{t+j} + \nu_{t+j} \frac{1}{\sigma} = 0$$

By deriving by $\pi_{t+j}$: 
$$-\pi_{t+j} + \psi_{t+j} = 0$$

By deriving by $x_{t+j}$: 
$$-\Gamma_x x_{t+j} + \nu_{t+j} - \kappa \psi_{t+j} = 0$$

So we get the following first order condition:

$$-\Gamma_x x_t + \sigma \Gamma_i i_t - \kappa \pi_t = 0$$

Or $x_t = -\frac{\kappa}{\Gamma_x} \pi_t + \sigma \frac{\Gamma_i}{\Gamma_x} i_t$

Here, $\psi_t$ is the Lagrangian multiplier associated with the inflation equation and $\gamma_t$ is the lagrangian multiplier associated with the IS curve. It should be noticed that $E_t \pi_{t+1}$ disappeared in this equation because the central bank doesn’t consider future dates.

Optimal paths:

$$\pi_t = D_{d1} u_t + D_{d2} g_t, \quad (1)$$

With $D_{d1} = \frac{\sigma^2 \Gamma_i (\rho - 1) - \Gamma_x}{A \kappa}$

$$A = -\kappa + \left( -\frac{\sigma \beta \rho^2}{\kappa} + (1 + \frac{\sigma}{\kappa} + \frac{\sigma \beta}{\kappa}) \rho - \frac{\sigma}{\kappa} \right) \sigma \Gamma_i + (\rho \beta - 1) \frac{\Gamma_x}{\kappa}$$

$$D_{d2} = -\frac{\sigma^2 \Gamma_i}{B}$$

$$B = -\kappa + \left( -\frac{\sigma \beta \lambda^2}{\kappa} + (1 + \frac{\sigma}{\kappa} + \frac{\sigma \beta}{\kappa}) \lambda - \frac{\sigma}{\kappa} \right) \sigma \Gamma_i + (\lambda \beta - 1) \frac{\Gamma_x}{\kappa}$$

$$x_t = D_{d3} u_t + D_{d4} g_t, \quad (2)$$

With $D_{d3} = \frac{\sigma^2 \Gamma_i (\rho - 1) - \Gamma_x (1 - \beta \rho) - 1}{A \kappa^2}$

$$D_{d4} = (\beta \lambda - 1) \frac{\sigma^2 \Gamma_i}{\kappa B}$$

$$i_t = D_{d5} u_t + D_{d6} g_t, \quad (3)$$
With \[ D_{d5} = \frac{\sigma^2 \Gamma_i (\rho - 1) - \Gamma_i}{A K} (\rho + \frac{\sigma(1-\beta \rho)}{\kappa}(\rho - 1)) + \frac{\sigma}{\kappa}(1-\rho) \]
\[ D_{d6} = \left( \frac{\sigma(\beta \lambda - 1)}{\kappa} (\lambda - 1) \right) \frac{\sigma^2 \Gamma_i}{B} + \sigma \]
with \[ u_t = \rho u_{t-1} + \eta_t \] and \( \eta_t \) is a white noise process with constant variance \( \sigma_u^2 \).

We can find also the real interest rate.

We get:

\[ r_t = \left( \frac{\sigma^2 \Gamma_i (\rho - 1) - \Gamma_i}{A K} \right) (\rho - 1) + \frac{\sigma}{\kappa}(1-\rho)u_t + \left( \frac{\sigma(\beta \lambda - 1)}{\kappa} (\lambda - 1) \right) \frac{\sigma^2 \Gamma_i}{B} + \sigma \] \( g_t \)

Money Path:

We incorporated \( \sigma_g \) into the path for interest rate to neutralize the shock from the demand curve. These paths are the optimal paths after a shock \( u_t \) from the Phillips curve. This shock \( u_t \) represents usually a difference between the marginal cost and the output gap.

In this process, deviations of inflation and output gap from steady state in the past are neglected. What counts is to bring back inflation and output back to target.

If the economy is subject to an impulse in the Phillips curve, contrary to inflation and output gap, price level is not forced to go back to previous levels, and goes up without limits. We can see that by deducting the path for the price level. We have \( p_t = \pi_t + p_{t-1} \) so we obtain:

\[ p_t = p_{t-1} + \frac{\sigma^2 \Gamma_i (\rho - 1) - \Gamma_i}{A K} u_t - \frac{\sigma^2 \Gamma_i}{B} g_t \]

Even if monetary policies use interest rate rules, money is not completely absent in the model. Then we can use the money demand equation to find the optimal path for money:

\[ m_t - p_t = \frac{\sigma}{b} y_t - \frac{1}{b} i_t + \omega_t \] \( (4) \)

So, using the paths previously shown

\[ p_t = \pi_t + p_{t-1} \]
\[ y_t = x_t + y_t \]

we get:

\[ m_t = m_{t-1} + \left[ \frac{\sigma}{b} \left( \frac{\sigma^2 \Gamma_i (\rho - 1) - \Gamma_i}{A K^2} (1 - \beta \rho) - \frac{1}{\kappa} \right) \right. \]
\[ - \frac{1}{b} \left( \frac{\sigma^2 \Gamma_i (\rho - 1) - \Gamma_i}{A K} (\rho + \frac{\sigma(1-\beta \rho)(\rho - 1)}{\kappa}) + \frac{\sigma^2 \Gamma_i}{B} \right) \] \( u_t \]
\[ - \frac{1}{b} \left( \frac{\sigma^2 \Gamma_i (\rho - 1) - \Gamma_i}{A K^2} (1 - \beta \rho) - \frac{1}{\kappa} \right) \]
\[ + \left[ \frac{\sigma}{b} (\beta \lambda - 1) \right] \frac{\Gamma_i}{\kappa B} \left( \frac{\sigma}{b} \left( \frac{\sigma^2 (\beta \lambda - 1)(\lambda - 1)}{\kappa} \right) \frac{\sigma^2 \Gamma_i}{B} + \sigma \right) \] \( g_t \]
\[ + \frac{\sigma}{b} (y_t - y_{t-1}) \] \( + \omega_t - \omega_{t-1} \)
Discretion case:

Figure 18:

This time the central bank commit to a path for not only current but also future inflation and output gap.

The objective is still to minimize the loss function

\[ E \left( \sum_{t=0}^{\infty} \beta^t \left( \pi_t + \Gamma_x x_t^2 + \Gamma_i i_t^2 \right) \right) \]

The central bank should minimize this loss function subject to the Phillips curve, or inflation equation. The IS curve doesn’t create any constraints on the choice of policy. It should thus maximize

\[ L = E \sum_{t=0}^{\infty} \beta^t \left( \frac{1}{2} (\pi_{t+1}^2 + \Gamma_x x_{t+1}^2 + \Gamma_i i_{t+1}^2) + \psi_{t+1} (x_{t+1} - x_{t+1}) + \frac{1}{\sigma} (i_{t+1} - \pi_{t+1}) - g_{t+1} \right) \]

\[ + \psi_{t+1} (\pi_{t+1} - \beta \pi_{t+1} - \kappa x_{t+1} - u_{t+1}) \]

The first order conditions are:

By deriving by interest rate:

\[ -\Gamma_i i_{t+1} + \psi_{t+1} \frac{1}{\sigma} = 0 \]

By deriving by inflation:

\[ -\frac{\psi_{t+1}}{\sigma \beta} - \psi_{t+1} - \psi_{t+1} + \psi_{t+1} = 0 \]

By deriving by output gap:

\[ -\frac{\psi_{t+1}}{\beta} - \psi_{t+1} - \psi_{t+1} + \kappa \psi_{t+1} = 0 \]

The first order conditions provide the following relationship:

\[ -\sigma \Gamma_i i_t + (\kappa + \sigma \beta + \sigma) \frac{\Gamma_i}{\beta} i_{t-1} - \frac{\sigma \Gamma_i}{\beta} i_{t-2} + \kappa \pi_i + \Gamma_x x_i - \Gamma_x x_{t-1} = 0 \]
In case of commitment:

Figure 19:

We can observe from figure 19 that the commitment policy with a social welfare incorporating volatility of interest rate implies a deflationary policy. In figure 20, we still have a better outcome with commitment than with discretion.

Figure 20:

5.2 Comparison of different policies

The idea is the same as in the previous section. The paths from the different alternative policies are used to calculate this new welfare. Analyzing the gain from the alternative policies, the result of this paper shows that the price level targeting rule still dominates the other policies. Even considering the volatility of interest rate, the benefit of emphasizing on bringing back price to its original level is predominant. However, it is the case when the central bank may choose the weight $\Gamma_p$ on output that appears on the loss function. The highest is price stickiness, the lowest this weight has to be to reach the highest welfare through price level targeting. If $\Gamma_p$ is not low enough, then nominal income targeting would be superior.
Figure 20: Violet is the discretion case when welfare includes volatility of interest rate.

Figure 21 shows the percentage gain of the different policies against each other. In the figure 21 d,
5.3 Volatility of interest rate and lower bound for interest rate issue

In a low inflation environment and when nominal interest rates are close to zero monetary policy makers would have a limited ability to ease in response to adverse shocks. The focus of the literature is to propose strategies when traditional monetary policies become apparently inefficient. An increasing number of economists revive the idea of stabilizing nominal GDP as a strategy. This paper has another perspective. Having found analytical solution to calculate welfare for different policies, it is also possible to develop formulas to assess the volatility of interest rate. As a consequence it is possible to evaluate to what extend the different policies can reduce this volatility to prevent the situation in which the nominal interest rate is immobilized at zero.

Figure 2 shows, for a specific level of price stickiness, how volatility evolve when central bank manipulate the weight $\Gamma$ on output that appears on different monetary policies. Using the calibration from Rotemberg and Woodford (1997), $\Gamma$ is considered to be around 0.0048 in the social welfare. This paper showed that the central bank can obtain the highest welfare when adopting a price level targeting rule with a very low $\Gamma_p$. According to the results shown in figure 21, it would require to increase that weight $\Gamma_p$ to reduce the volatility of interest rate. However, it can be seen that that the central bank could be significantly more effective at lower that volatility by choosing the discretion policy with the social welfare incorporating transaction frictions. As shown in figure 20 and 21, it would have for consequence to reduce welfare. It is the cost of avoiding the problems created by the lower bound for interest rate.

Figure 2:

6 Conclusion

As emphasized by Clarida, Gali, and Gertler (1999), Rotemberg and Woodford (1997) and explained by Woodford in “Interest and Prices” (2003), inertia is the key element to reach optimality in monetary policy. Such behavior increases welfare by improving the inflation-output gap trade-off. Maximizing social welfare, a commitment rule would provide this kind of benefit when compared to a discretion policy. However, some authors suggested different “discretion” policies to reproduce the inertial behavior observed in a commitment policy. In the speed limit policy, the central bank would focus on the growth of output gap instead of its level. For the price level targeting strategy, the emphasis would be on price level rather than inflation. Finally, the nominal income growth would replace output gap by the growth of output gap plus inflation.

The results presented in this paper show that the price level targeting would be a superior policy when the model is a basic New Keynesian model with forward Phillips and IS curves. For any price stickiness, the central bank could select a weight $\Gamma_p$ on output to get a welfare that would be above a welfare obtained by the other policies. However, reaching those levels comes at the price of having a high volatility of interest rate. In the case the interest rate reaches the lower bound, the central bank may need to increase the weight
on output but it would probably not be enough. Policy makers would have to change policy and adopt an inflation targeting strategy with a welfare objective that includes the volatility of interest rate. Furthermore, those policies are considered as discretion regimes but it is still a matter of discussion. They are derived as if expectations were not included but the paths adopted are similar to the ones found with the commitment rule and the central bank may be facing the same kind of incentive to cheat and switch policy.

References


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