Reconstructing the Great Recession*

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Abstract

This paper evaluates the role of the construction sector in accounting for the performance of the U.S. economy in the past decade. During the Great Recession (2008-09), employment in the construction sector experienced an unprecedented decline that followed the largest expansion in employment since the 1950s. Despite the small size of the construction sector, its interlinkages with other sectors in the economy propagate the effect from changes in the demand for residential investment, hence amplifying the effect on the overall economy. An input-output analysis reveals that the construction sector has been an important driver of the dynamics of aggregate employment and output of the U.S. economy through the sectoral interlinkages. The importance of interlinkages is illustrated in a static model and then quantified in a generalized framework that includes fixed and residential investment. The model is calibrated to reproduce the boom-bust dynamics of construction employment in the 2000-10 period. We find that construction and its interlinkages account for a large share of the actual changes in aggregate employment and gross domestic product during the expansion and the recession. Despite it is simulated with a shock to the demand of housing, the recession in our model looks, through the lens of business cycle accounting, as generated by a worsening of the labor wedge.

Keywords: Residential investment, multisector models, business cycle accounting

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1 Introduction

Several U.S. macroeconomic aggregates, such as employment and gross domestic product (GDP), have yet to return to pre-Great Recession trends. While there is still no consensus on why the recovery of the U.S. economy has been slow compared with a traditional postwar recession, some point to the role of the construction sector.\(^1\) This sector collapsed as the real estate bubble burst in 2007, an event commonly believed to be the key factor underlying the U.S. Great Recession and the poor economic performance that followed. Construction generally contributes about 5 percent of employment and output, but in this particular episode, it had an unprecedented decline following the largest expansion in employment since the 1950s.

Why the construction sector collapsed is unclear for the simple reason that what created the boom in the first place is also unclear. Be that as it may, there is general agreement that the bursting of home values did translate into a sudden drop in the demand for a specific asset in which American households had, until then, invested a substantial portion of their wealth: residential housing. The analysis starts from this (hopefully) incontrovertible fact and studies the contribution of the construction sector to U.S. economic growth, particularly during the construction boom and the Great Recession.

The macroeconomic impact of the construction sector is derived from its interlinkages with other productive sectors of the economy. These linkages propagate the effect of a decline in demand for residential investment to the rest of the economy. This paper argues that the construction sector has been an important driver of the dynamics of aggregate employment and output in the U.S. economy through sectoral interlinkages.

A simple way to analyze the contribution of the construction sector to the evolution of these aggregate dynamics and, in particular, the slow recovery, is to use input-output tables. Using the requirement matrices allows comparison of the evolution of U.S. employment and gross output with an artificial economy where the construction sector has been completely eliminated.\(^2\) The difference between these paths can be attributed to the construction sector. The elimination of the construction sector implies a lower demand for productive inputs from other sectors. These other sectors in the economy would have to reduce their respective requirements of productive inputs and so forth. Figure 1 clearly illustrates, in fact, that the dynamics of employment and gross output for these two economies are very different. For the U.S. economy (including construction) total employment increased about 6 percent between 2002 and 2006. All that employment growth was lost during the Great Recession and continued to decline in 2010. In contrast, the economy without a construction sector

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\(^1\)Leamer (2007), who argues that since World War II the U.S. economy has had eight recessions preceded by substantial problems in housing and consumer durables.

\(^2\)For each year available in the Bureau of Economic Analysis (BEA) and Bureau of Labor Statistics (BLS) requirement tables. The effect of a shock to the construction sector of the size of its total output was computed using the corresponding table to calculate the impact of the construction industry on total gross output and total employment. Those values were then removed, together with the values of the construction industry per se, from the aggregate gross output and value added of the U.S. economy, year by year. In the figure construction includes real estate and leasing.
has a slower recovery from the 2001 recession but rapid employment growth in 2005. This artificial economy does not start the employment destruction until 2008, and the magnitude of the employment decline is half that of the other economy. In addition, unlike the economy with construction, the recovery starts in 2010. This exercise shows that the construction sector was a clear factor contributing to employment growth between 2002 and 2005 and has dragged since the Great Recession. A similar conclusion can be drawn by analyzing the series for gross output.

**Figure 1: Contribution of the Construction Sector to the Dynamics of Employment and Gross Output**

Our empirical analysis shows the role of sectoral interlinkages in propagating changes in housing demand to other sectors of the economy. A simple accounting input-output framework reveals that construction accounts for 52 percent of the decline in employment and 35 percent of the decline in output. The input-output model is static and does not allow for movement in relative prices. To highlight the functioning of the paper’s main mechanism we construct a simple (static) multisector model. The model reveals that the effects of a decline in housing demand are amplified in the presence of sectoral interlinkages and when housing is complementary to other goods. In the absence of these mechanisms, the model suggests that a decline in construction should not propagate to other sectors.

To assess the quantitative importance of sectoral interlinkages, we construct a dynamic multisector model with residential investment and interconnected sectors. The presence of irreversibility constraints introduces an asymmetry between expansions and the recessions.

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3 The methodology in our paper is close to that of Davis and Heathcote (2005). They study the dynamics of residential investment and house prices in a multisector business cycle model. Their model is successful in reproducing the volatility of residential investment, but they do not focus on the propagation of shocks due to sectoral interlinkages.

4 The analysis abstracts from both the increase in the burden of debt brought about by the decline in home prices (which is the focus of Garriga, Manuelli, and Peralta Alva, 2012a) and the reduction in credit activity it implied, two factors that are likely to have played (and may still be playing) a major role in the
During the expansion, the increasing housing demand results in uncharacteristically high GDP levels and growth, driven by extremely high levels of output in the construction and housing sector and in all related activities. Growth is constrained only by the accumulation of capital and residential structures. The decline in housing demand stops economic growth and the inflated levels of economic activity collapse. As a direct impact, this leaves the economy with (potentially) a surplus of equipment (residential structures) appropriate for housing-related activities but not easily transferable to other productive activities. The relatively low depreciation of residential structures implies that home values, the construction sector, and aggregate consumption and investment should take a long time to recover. When the irreversibility constraint binds for an extended period, the asymmetric effect between booms and busts is magnified. In addition, sectoral linkages interact with the final demand for goods, other than housing, in the case of complementarities (i.e., durables, housing-related expenditures). Hence, a decline in the demand for housing indirectly reduces the demand of the complementary goods and, thus, the output from these sectors. As a result, the magnitude of the impact on output and total employment can be amplified when this additional mechanism operates.

To quantify the contribution of construction to the overall economy, we calibrated a sequence of demand shifters in consumer preferences to match the dynamics of employment in construction for different model specifications with and without interlinkages. Qualitatively the effects are similar in these economies, but quantitatively, the magnitude of the boom-bust cycle in total employment and output is substantially larger with sectoral interlinkages. During the boom, both sectors expand and contribute to the growth of output and employment by 2 percent and 2.5 percent, respectively. During the housing bust, the magnitude depends on the number of periods the irreversibility constraint binds. This amplifies the asymmetric response between the boom and the bust episode. When this effect is quantitatively important, the decline in output is 3.3 percent and in employment is 3.8 percent. For a lower degree of complementarity, the asymmetric effect is not as large but still significant. To identify the role of interlinkages, we perform two exercises to control for the contribution of intermediate goods during the construction boom and bust. In those cases, changes in housing demand consistent with the dynamics in construction employment play only a small role in macroeconomic quantities even when the complementarity between consumption goods and housing services is high.

In the model, all changes in employment and output are generated by variations in housing demand. We can calculate the contribution of construction observed in the data by comparing the magnitudes generated by the model. During the expansion between 2002 and 2007, the construction sector accounts for a significant share of growth of employment (between 29 percent and 61 percent) and GDP (between 8 percent and 15 percent). More overall process. Although these factors could interact with the sectoral interlinkages, abstracting from them captures the contribution of the real side of the economy in the recession. In the model, a decline in the demand for homes generates a readjustment of the portfolio and a decline in the demand for intermediate inputs. The lower demand of intermediate goods deprives the real side of the economy and generates a significant decline in employment and real activity.
importantly, its contribution during the Great Recession was between 28 percent and 43 percent of employment and 43 percent and 60 percent of GDP.

The burst of the real estate “bubble” might have substantially lowered potential output and created a substantial “displacement effect” for both labor and capital that may take quite some time to fully absorb. Some researchers have referred to this displacement effect as a worsening of the labor frictions. For example, Arellano, Bai, and Kehoe (2010) and Ohanian and Raffo (2012) attribute to this factor, most of the recession, whereas total factor productivity has been less important. Since the model captures a significant decline in employment and output, we also perform a business cycle accounting exercise on simulated data from the model. The model does not have any distortions, but the intersectoral linkages, movements on relative prices across sectors, and the effects on the demand for labor input can be interpreted as distortions through the lens of the one-sector neoclassical growth model. This methodology attributes the recession in our model to the labor wedge. The magnitude of the worsening of the labor wedge is about 62 percent of the total change observed in the data. Importantly, in both our model and the data the worsening is due to the consumer side of the labor wedge and not to differences between wages and the marginal product of labor.

Note that the causal links discussed here operate in an environment not subject to the market failures and price-adjustment frictions now standard in business cycle models used to guide fiscal and mainly monetary policy. In the language of those models, ours is a model of potential output in which fluctuations of economic activity cannot be counteracted with standard policy tools. Our findings are important for policy discussions using those models because they imply that output gaps may not be as large as previously thought.

The contribution of the construction sector cannot fully account for the dynamics of employment and output since 2002. Other relevant factors not incorporated in the analysis are certainly important. Many suggest (i.e., Black, 1995; Hall, 2011; and Kocherlakota, 2012) that high interest rates could be responsible for the slow recovery. These authors argue that even in models with perfect competition and price flexibility (i.e., lacking the typical frictions in business cycle models), too-high levels of interest rates may result in substantially lower levels of output and employment. Since some interest rates appear to be currently constrained by the zero lower bound, such analyses appear particularly pertinent. Others argue that the level of uncertainty (i.e., Bloom, 2009; Arellano, Bai, and Kehoe, 2010), government policies (i.e., Herkenhoff and Ohanian, 2012a), and excessive debt overhang in the economy (i.e., Garriga, Manuelli, and Peralta-Alva, 2012a; Kehoe, Ruhl, and Steinberg, 2012; and Herkenhoff and Ohanian, 2012b) may be responsible for the lackluster recovery.

The remainder of the paper is organized as follows. In the next section, we present some empirical evidence and perform standard calculations using the input-output matrix of the U.S. economy. Section 3 presents a simple static model of interdependence that is used to illustrate the key mechanism at work in the analysis. Section 4 presents the quantitative model. Section 5 reports the numerical results and the robustness analysis. Section 6 compares the implications of the model in terms of business cycle accounting
methodology, and Section 7 offers some concluding comments.

2 Empirical Analysis

In this section, we explore the role of construction in the U.S. in the past decade. In the analysis, the definition of the construction sector does not include real estate and leasing. First, we document the decline in construction output and employment and its effects on the aggregate economy. Second, we use input-output tables to describe the interlinkages of the construction sector with the rest of the economy and then use the input-output model to estimate the macroeconomic effects of a decline in construction.

2.1 Construction during the Great Recession

The construction industry went into recession 18 months before the overall economy. Employment fell from 7.7 million in 2006:Q3 to 5.5 million in 2011:Q1; it still has not emerged from the trough and the decrease may well continue in the future. Figure 2 shows that employment, gross output, and GDP in the construction sector dropped about 30 percent during this period, with the largest year-to-year decline between 2008 and 2009.

Figure 2: The Construction Sector during the Great Recession

![Figure 2: The Construction Sector during the Great Recession](image)

Source: BEA.

The share of construction in total employment and total GDP is about 5 percent, as shown by Figure 3. During the Great Recession the share of construction decreased by 1.2 and 1.5 percent in total GDP and total employment, respectively. The fact that the employment share on the figure appears to be higher than the one for GDP share reflects the fact that construction is relatively more labor intensive than the rest of the economy, which is just another way of saying that average labor productivity in construction is below the economy-
wide mean.\textsuperscript{5}

Figure 3: Construction in the U.S. Economy
Share of Construction in the Economy  Spending and Residential Investment

The dynamics of the demand for construction can be seen in the behavior of private residential investment and total spending on construction. The right panel in Figure 3 shows that both series have an abrupt decline between 2006 and 2010, while the extent of their growth between roughly 1990 and 2006 shows that the real estate boom of those years was exceptional by historical standards.

2.2 Construction in an Interlinked Economy

According to the National Bureau of Economic Analysis (NBER), the Great Recession started in 2007:Q4 and lasted until 2009:Q2. During this period total employment decreased abruptly, by roughly 8 million jobs. Table 1 shows that the drop in construction employment and value added account for about 27 and 22 percent of the total, respectively. Here the initial period is the average of the annual data for 2006 and 2007 and the final period is the year 2009. Larger shares are obtained if this computation is performed using the peak

\textsuperscript{5}The time series reported in Figure 2 also suggests that the distance from the average is procyclical, but this is largely irrelevant to our investigation.
(2006:Q1) and the trough (2010:Q4) of the construction series.

**Table 1: Direct Impact of Construction on Employment and Income**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Peak 2006-07</th>
<th>Trough 2009</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construction employment (millions)</td>
<td>7.7</td>
<td>6.0</td>
<td>-1.6</td>
</tr>
<tr>
<td>Total employment (millions)</td>
<td>136.8</td>
<td>130.8</td>
<td>-6.1</td>
</tr>
<tr>
<td>Share of drop accounted for by construction (%)</td>
<td>27.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Construction income ($ billions)</td>
<td>662.8</td>
<td>493.7</td>
<td>-169.1</td>
</tr>
<tr>
<td>Total income ($ billions)</td>
<td>11,664</td>
<td>10,896</td>
<td>-768.7</td>
</tr>
<tr>
<td>Share of drop accounted for by construction (%)</td>
<td>22.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: Authors’ calculations using data from the BEA.

These numbers could be conservative because real estate and leasing are not included as part of construction. Expanding the definition of the construction sector to include real estate and leasing increases the magnitudes described in Table 1. In this case, construction could account for a 39 percent of the decline in employment and 47 percent of the drop in value added.⁶

Note that the contribution of the construction sector to the overall decline in employment and output reported in Table 1 is the direct effect only. Additional evidence suggests that the total impact of the drop in the construction sector on the overall economy may have been even larger. Figure 4 depicts the share of the gross output of each industry purchased as intermediate inputs by the construction industry.

**Figure 4: Construction Purchases from Other Sectors**

Source: 2006 Use matrix from the BEA input-output tables.

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⁶The additional calculations are available upon request.
One way to interpret the data is to imagine a total collapse of the construction industry. In that case, construction would buy nothing from other industries and the numbers in Figure 4 would represent the percentage drop in each industry’s output after such an event while abstracting from interlinkages between those sectors. For instance, gross output in both manufacturing and retail would fall by roughly 7.6 percent. To understand the aggregate effects of a decline in the final demand for construction it is necessary to work through all the interlinkages. Initially, the construction sector will demand less from other sectors. As a result, these other sectors will decline their input requirements from the sectors from which they purchase goods. When large sectors such as manufacturing, retail, or professional businesses scale down, there are additional indirect effects on the construction sector. By using the requirement matrices from the BEA input-output tables, it is possible to calculate all the interactions and feedback effects of changes in the demand for a specific sector and the aggregate economy. Use of this matrices makes possible to explore the role of sectoral linkages in accounting for the Great Recession.

The input-output tables provide a framework to measure the contribution of the construction sector to the aggregate decline in economic activity during the Great Recession. To isolate the effect of construction from changes in other sectors we compute the decline in gross output associated with the observed decline in the construction sector between 2006-07 (average of these years) and 2009. The same procedure is used to calculate the impact on employment. The choice of the period we analyze is rather conservative: The share of the reduction in sectoral output mechanically explained by the drop in construction could be substantially larger if the difference between the peak and trough of construction output were used instead. The blue bars in Figure 5 represent the percent change of gross output

\footnote{In this case, the employment requirement matrix from the Bureau of Labor Statistics (BLS) is used.}
As Figure 5 shows, construction and total gross output declined by 21.3 percent and 6.2 percent, respectively. Employment in construction decreased by roughly the same amount as gross output, 21.5 percent, while total employment declined by 4.4 percent during the same period. The red striped bars in Figure 5 represent the decline attributed to the construction sector. For example, construction accounts for a significant part of the gross output decline in mining, about 68 percent of it, while it accounts for little, about 6 percent, of the decline in entertainment, recreation, and accommodation and food services. According to this exercise, construction is capable of accounting for about 35 percent of the total decline in gross output and about 52 percent of the total decline in employment. These numbers contrast with the direct impact estimates that account for 27 percent of the decline in employment and 22 percent of gross output. The difference between the direct and total effect of construction is due to the magnifying role of the interlinkages.

Another important observation from Figure 1 is how the slow recovery in the construction sector is dragging down the overall economy. The recoveries for gross output and total employment were about 34 percent (5.4 percent vs. 3.6 percent) and 70 percent (1.4 percent vs. 0.4 percent) slower, respectively. The contribution of construction to the slow recovery can be further decomposed by performing an input-output exercise similar to that in Figure 5. Figure 6 shows how the different sectors would look like if construction were growing at
pre-recession levels (or at 2004 levels).

**Figure 6: Sectoral Changes during the Recovery**
(Data and Input-Output Simulation)

<table>
<thead>
<tr>
<th>Industry</th>
<th>Employment, % Change (BLS Requirement Matrix)</th>
<th>Output, % Change (BEA Requirement Matrix)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Other Services</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leisure &amp; Hosp.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Educ. &amp; Health</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prof. &amp; Bus.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Financial</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Information</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trans. &amp; Ware.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Retail</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wholesale</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Manufacturing</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Construction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Utilities</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mining</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: Authors' calculations using BEA and BLS data.

The blue bars in Figure 6 display the actual change of gross output and employment in 13 industries and the total economy. While the data show that gross output increased by 2.4 percent in this period, employment actually continued to decrease by 0.7 percent. If construction were growing at its pre-recession levels, total gross output would have increased by more than 5 percent in the 2009-10 recovery and employment would have still decreased, but at the lower rate of 0.12 percent. The industries that would have grown the most in terms of gross output in this scenario are wholesale trade (23 percent), retail and mining (15 percent), and transportation and warehousing (9.5 percent).

### 3 Simple Model of Interlinkages

This section presents a simple model to illustrate the transmission mechanism from the construction sector to the rest of the economy. As a result, the demand of inputs used to produce homes falls, but the other sectors also decline from the reduced housing demand. In the model presented in this section, most of the action comes from the productive structure. The effects from this model will be operative in the quantitative model, but it is useful to differentiate the short-run effects from the dynamic effects due to changes in the accumulation of reproducible inputs (i.e., capital and housing structures).\(^8\)

\(^8\)The Appendix includes a generalized version where a decline in the demand for housing can also affect the demand for other goods.
3.1 Model without Interlinkages

Consider a two-sector economy with a representative consumer. Individual preferences are defined over consumption, \( c \); housing, \( h \); and labor, \( n \). The utility index is given by \( u(c, h, n) = \log c + \theta \log h - an \). Goods are produced in different sectors (consumption \((y)\) and construction \((s)\)). In each sector, competitive firms have access to a linear technology that uses labor input to produce each good. The technology to produce goods is \( c = A^y n^y \), and the technology to produce construction is \( h = A^s n^s \), where \( A^j \) represents the productivity of sector \( j \). The total labor allocated across sectors satisfies \( n^s + n^y = n \).

A competitive equilibrium in this economy is an allocation \( \{c, h, n\} \) and prices \( \{w, p^h\} \) that satisfy the following: i) the optimization problem of the consumer, ii) maximization of firms’ profits, and iii) clearing of markets. Since the competitive equilibrium is efficient, it is simpler to solve for the social planner problem. The planner solves for the allocation of labor inputs across sectors and the level of employment,

\[
\max_{n, n^s} \{ \log A^y (n - n^s) n^y + \theta \log A^s n^s - an \}.
\]

The optimal solution allocates a constant fraction of labor in every sector \( n^s = \theta/a \), and \( n^y = 1/a \). Given the employment level in each sector, the optimal consumption of goods and housing is \( h = A^s \theta/a \) and \( c = A^y/a \). The relative price of housing (construction) is simply the ratio of marginal utilities evaluated at the optimal level of consumption and housing,

\[
p^h = \frac{\theta c}{h} = \frac{A^y}{A^s}.
\]

Total value added depends on the level of employment, productivity in the goods sector, and the relative demand for housing,

\[
VA = c + p^h h = A^y \frac{1}{a} + \left( \frac{A^y}{A^s} \right) A^s \frac{\theta}{a} = \left( \frac{A^y}{a} \right) (1 + \theta).
\]

In this model, changes in the demand for housing have positive effects in value added:

\[
\frac{\partial VA}{\partial \theta} = \left( \frac{A^y}{a} \right) > 0.
\]

However, these effects operate only via the housing sector. The goods sector does not respond by increasing employment. This result does not hold in an economy where the productive sectors are interconnected.

3.2 Model with Interlinkages

With production interlinkages, some output of the goods sector is used as an input to produce homes, \( m^y \), and some output of the housing sector is used to produce goods, \( m^s \). To keep
the structure simple and tractable, we assume Leontief production functions in each sector. With interlinkages, the social planner solves

$$\begin{align*}
\max_{n^s, n^y} & \{ \log c + \theta \log h - a(n^s + n^y) \}, \\
\text{s.t.} & \quad c + m^y = A^y \min \{ n^y, \frac{m^s}{\varepsilon^y} \}, \\
& \quad h + m^s = A^s \min \{ n^s, \frac{m^y}{\varepsilon^s} \},
\end{align*}$$

where the parameters $\varepsilon^y$ and $\varepsilon^s$ represent the relative intensity of the intermediate input. As $\varepsilon^j \to 0$, the quantity of the intermediate good required converges to zero, $m^j \to 0$. This formulation encompasses the previous model as a special case. The optimal employment and labor allocation across sectors yields

$$\begin{align*}
\bar{n} &= \frac{1 + \theta}{a}, \\
\bar{n}^s &= \frac{1}{a} \left( \frac{(A^y + \varepsilon^s)\varepsilon^y + \theta (A^s + \varepsilon^y)A^y}{(A^y + \varepsilon^s)(A^s + \varepsilon^y)} \right), \\
\bar{n}^y &= \frac{1}{a} \left( \frac{(A^y + \varepsilon^s)A^s + \theta (A^s + \varepsilon^y)\varepsilon^s}{(A^y + \varepsilon^s)(A^s + \varepsilon^y)} \right).
\end{align*}$$

Notice that total employment is the same as in the model with no interlinkages. However, the labor demands in each sector are interconnected. Now, a change in the demand for housing requires goods from the other sectors as well. The size of the increase depends on the specific parameters of the production function.

Replacing the optimal decisions in the resource constraint gives the production of final goods and housing,

$$\begin{align*}
c &= \frac{1}{a} \frac{(A^y A^s - \varepsilon^y \varepsilon^s)}{(A^s + \varepsilon^y)}, \\
h &= \frac{1}{a} \frac{\theta (A^y A^s - \varepsilon^y \varepsilon^s)}{(A^y + \varepsilon^s)}.
\end{align*}$$

The relative price of construction now also depends on the Leontief coefficients,

$$p^h = \frac{(A^y + \varepsilon^s)}{(A^s + \varepsilon^y)}.$$ 

With interlinkages, total value added is

$$VA_{\text{interlinkages}} = \frac{(1 + \theta)}{a} \left[ \frac{(A^y A^s - \varepsilon^y \varepsilon^s)}{(A^s + \varepsilon^y)} \right].$$
The change in value added with the housing shock is

\[
\frac{\partial VA}{\partial \theta} = \left( \frac{A^y}{a} \right) \left[ \frac{A^y A^s - \varepsilon^y \varepsilon^n}{A^y A^s + A^y \varepsilon^n} \right].
\]

### 3.3 The Role of Interlinkages

The goal of this section is to illustrate why considering interlinkages is key for the amplification of shocks to the construction sector. Notice that for this result it is not necessary to assume complementarity in consumption between \( c \) and \( h \). The amplification generated by interlinkages is magnified with the addition of complementarity.

In order to gain insights on the quantitative results, we now perform an exercise analogous to the one in the quantitative section with the two models presented above. Imagine we want to reproduce a drop in employment in the construction sector by a change in the demand for housing via \( \theta \):

\[
n^s = \frac{\varepsilon^y}{a(A^s + \varepsilon^n)} + \frac{\theta}{a} \frac{A^y}{A^s + \varepsilon^n}.
\]

The demand from the construction sector has two components: The first component depends positively on the relative importance of structures as intermediate in the production of consumption goods, \( \varepsilon^y \); and the second component depends positively on the demand of houses, \( \theta \), and negatively on the relative intensity of nonhousing intermediate goods in the production of housing.

Given the expression, a change in the level of employment as a response to changes in \( \theta \) when \( \varepsilon^s > 0 \) (the construction sector demands intermediate goods inputs from the rest of the economy) is given by

\[
\left( \frac{\partial n^s}{\partial \theta} \right) = \frac{1}{a} > \frac{1}{a} \left( \frac{\tilde{A}^y}{\tilde{A}^y + \varepsilon^n} \right) = \left( \frac{\partial n^s}{\partial \theta} \right)_{\text{interlinkages}}^\text{interlinkages} < 1.
\]

In this case, the change in \( \theta \) generates a smaller decline in construction employment in the model with interlinkages, independently of the value of \( \tilde{A} \) (this notation allows the possibility that both economies are calibrated to different levels of productivity). To generate the same decline in the level of employment in both economies, a larger change in \( \theta \) is required in the economy with interlinkages.

Recall that the change in total employment is the same in the two models presented above:

\[
\frac{\partial n^s}{\partial \theta} = \left( \frac{\partial n^s}{\partial \theta} \right)_{\text{interlinkages}} = \frac{1}{a}.
\]

Therefore, the larger change in \( \theta \) in the model with interlinkages implies a larger change in total employment.
Alternatively, it is direct to calculate the effect from a proportional change of 50 percent in the employment of the construction sector, $n_s$, in total employment. The baseline model (without interlinkages) requires the demand parameter for housing to drop by half, $\theta_0 = \theta_0 / 2$. The model with interlinkages requires a larger drop in $\theta$ when the goods sector uses structures as intermediates in production, $\varepsilon_y > 0$. Formally, $\theta^{\text{interlinkages}}_1 = \theta_0 / x$, where

$$x = 2 \frac{\theta (A^s + \varepsilon_y) A^y}{\theta (A^s + \varepsilon_y) A^y - (A^y + \varepsilon_y) \varepsilon_y} > 2.$$  

In general, the stronger the role of interlinkages (high values of $\varepsilon_s, \varepsilon_y$), the larger the decline in total employment generated with a shock $\theta$ that reproduces the same decline in construction employment.

4 Quantitative Model

The previous models are static and very stylized and, although they may give reasonable predictions in the very short run (i.e., on impact of a given shock or change in relative prices), they may also become less reasonable when the time horizon is increased (medium and long-run). Simulating the path of an economy over time by linking a sequence of static input-output matrices does not allow taking into account either the general equilibrium or the intertemporal effects of a given shock to initial conditions. The model presented in this section is a generalization of the static model and allows for both richer dynamics and a certain degree of substitution between productive inputs. The dynamic model introduces an asymmetry between the expansion and the recession due to the irreversibility constraints. This effect can be amplified in the presence of interlinkages.

4.1 Households

The total population size, $N_t$, is normalized to 1. Household preferences are defined by a time-separable utility function, $u(c_t, h_t) + \theta_t h_t + \gamma v(1 - n_t)$, where $c_t$ represents consumption goods, $h_t$ represents housing services, $n_t$ represents labor supplied in the market, and $\gamma > 0$ represents the relative weight of leisure in preferences. Housing provides two sources of utility; one is complementary to consumption goods and the other is linear by itself. This reduced form captures the intrinsic desire to consume homes and in the pricing equation for housing services appears as a bubble term. This is the term that will be responsible for the movements in housing demand that ultimately propagate to the construction sector and the overall economy via the interaction across sectors. The utility functions $u$ and $v$ satisfy the usual properties of differentiability and concavity. The sequence of utilities is discounted by the term $\beta \in (0, 1)$. Housing services are produced according to a technology, $H(s_t, l_t)$, that combines the physical structure, $s_t$, and land, $l_t$. The technology has constant returns to scale and satisfies $H'_i > 0$, $H''_i < 0$, and $H''_{ij} > 0$. Housing structures depreciate at a constant rate,
In each period, the numeraire is the spot price of the manufacturing good. The household also invests in structures used by firms. These are rented as an input in the production function (i.e., capital and labor). All investment decisions are subject to an irreversibility constraint and have different depreciation rates. These are relevant to generate asymmetric effects between the booms and the recessions. Formally, the representative consumer chooses \( \{c_t, h_t, n_t, k_{t+1}, s_{t+1}, l_{t+1}\}_{t=0}^{\infty} \) to maximize

\[
\max \sum_{t=0}^{\infty} \beta^t [u(c_t, h_t) + \theta_t h_t + v(1 - n_t)],
\]

subject to

\[
c_t + x_t^k + p_s^t x_t^s = w_t n_t + r_t^k k_t + p_l^t(l_t - l_{t+1}) + \pi_t,
\]

\[
h_t = H(s_t, l_t),
\]

\[
x_t^k = k_{t+1} - (1 - \delta_k) k_t \geq 0,
\]

\[
x_t^s = s_{t+1} - (1 - \delta_s) s_t \geq 0,
\]

together with the transversality condition and no-Ponzi-game conditions. The prices are defined as follows: \( p_s^t \) is the price of infrastructure, \( p_l^t \) is the price of land, \( w_t \) represents the wage rate, and \( r_t \) is the gross return on capital. To facilitate computing the rental rate for housing services, our specification allows land trading, \( l_t \), even if in equilibrium there is no trading of land, which is owned by the representative household and inelastically supplied. The term \( \pi_t \) represents profits from the productive sector.

The relevant first-order conditions of the consumer problem are

\[
\frac{\gamma v'(1 - n_t)}{u_c(c_t, h_t)} = w_t, \forall t,
\]

\[
\frac{u_c(c_t, h_t)}{\beta u_c(c_{t+1}, h_{t+1})} = 1 + r_{t+1}^k - \delta_k, \forall t,
\]

when the irreversibility constraints do not bind, \( x_t^k > 0 \). The relevant conditions for housing decisions for the case with positive housing investment \( (x_t^s > 0) \) satisfy

\[
\frac{u_h(c_t, h_t) + \theta_t}{u_c(c_t, h_t)} = R_t, \quad \forall t,
\]

\[
p_s^t = \frac{1}{1 + r_{t+1}^k} \left[ R_{t+1}^s H_s(s_t, l_t) + p_s^t (1 - \delta_s) \right],
\]

\[
p_l^t = \frac{1}{1 + r_{t+1}^l} \left[ R_{t+1}^l H_l(s_t, l_t) + p_l^t \right],
\]

where \( R_t \) represents the implicit rental price for housing services measured in terms of consumption units at time \( t \). The model predicts no arbitrage between land investment and housing capital. The last expressions state that the current cost of purchasing a unit of housing structures (land) equals the future return of housing services derived from the hous-
ing capital (land) valued at market prices and the capitalization.

4.2 Manufacturing Sector

The production sectors incorporate the simplest input-output structure. To operate, each sector requires, among other things, the output of itself and of the other sector as intermediate inputs. To capture this fact, we deviate from common practice and write all production functions in terms of gross (as opposed to net, i.e., value-added) output, at least initially. Capital goods, which are produced in the manufacturing sector, must be distinguished from the intermediate inputs from the same sector since they last more than one period. In the baseline model, capital goods (physical capital and business structures) are used in the manufacturing sector. Both investments satisfy the putty-clay assumption on sector-specific investment.

Formally, let \( m_{i,j}^y \) be the intermediate input produced by sector \( i \) and used by sector \( j \). The manufacturing sector operates in a competitive market and uses a technology that combines \( Y_t = A_t^y F(k_t, n_t^y, m_t^{y,y}, m_t^{s,y}) \) to produce its gross output:

\[
Y_t = c_t + x_t + m_t^{y,y} + m_t^{y,s}.
\]

The production function \( F \) is a constant returns to scale Cobb-Douglass function. The firm’s optimization problem is

\[
\pi_t^y = \max_{k_t, n_t^y, m_t^{y,y}, m_t^{s,y}} Y_t - w_t n_t^y - r_t^k k_t - m_t^{y,y} - p_t^s m_t^{s,y}, \quad \forall t,
\]

\[
s.t. \quad Y_t = A_t^y F(k_t, n_t^y, m_t^{y,y}, m_t^{s,y}), \quad \forall t,
\]

where the price of manufacturing’s output is normalized to 1. The constant returns to scale assumption implies zero equilibrium profits, \( \pi_t^y = 0 \), and marginal cost pricing for each input

\[
r_t^k = A_t^y F_1(k_t, n_t^y, m_t^{y,y}, m_t^{s,y}),
\]

\[
w_t = A_t^y F_2(k_t, n_t^y, m_t^{y,y}, m_t^{s,y}),
\]

\[
1 = A_t^y F_4(k_t, n_t^y, m_t^{y,y}, m_t^{s,y}),
\]

\[
p_t^s = A_t^y F_5(k_t, n_t^y, m_t^{y,y}, m_t^{s,y}).
\]

4.3 Construction Sector

The construction sector is also competitive: Its net output consists of residential structures, purchased by the households, while its gross output also includes structures used as an intermediate input in both production sectors. In the baseline case, purely for simplicity, we assume this sector has a fixed stock of capital; hence its value added is split between the wages of labor and the rent accruing to the owner of the fixed capital stock (the representative households). Implicit in this formulation is a somewhat extreme assumption about the
mobility of factors from one sector to another: While labor can move freely, the stock of capital invested in the construction sector is completely immobile (either way), and variations in investment activity have an impact on only the manufacturing sector. The technology for gross output,

\[ X_t^S = x_t^s + m_t^{s,s} + m_t^{y,y} = A_t^s G(n_t^s, m_t^{s,s}, m_t^{y,y}), \]

therefore exhibits decreasing returns to scale in labor and the intermediate input mix. In the benchmark economy, we assume \( G(\cdot) \) is a constant elasticity of substitution (CES) and the intermediate inputs aggregator is Cobb-Douglass. The optimization problem of the representative firm is now

\[
\pi_t^s = \max_{n_t^s, m_t^{s,s}, m_t^{y,y}} p_t^s X_t^S - w_t n_t^s - p_t^{s,s} m_t^{s,s} - p_t^{y,y} m_t^{y,y}, \quad \forall t,
\]

s.t. \( X_t^S = A_t^s G(n_t^s, m_t^{s,s}, m_t^{y,y}), \quad \forall t. \)

The first-order conditions are similar to those of the representative firm in the manufacturing sector and are not repeated here. Note that because of the presence of a fixed stock of capital, firms’ profits are not zero in equilibrium in this sector. It is worth emphasizing that \( p_t^s \) reflects the cost of producing new structures. The equilibrium price of a house differs from this value since it depends on the relative value of structures and land.

### 4.4 Competitive Equilibrium

The notion of competitive equilibrium is completely standard.

**Competitive Equilibrium:** Given a sequence of values \( \{A_t^y, A_t^s, \theta_t\}_{t=0}^\infty \), a competitive equilibrium consists of allocations \( \{c_t, x_t^k, x_t^{s,s}, x_t^{y,y}, l_t^s, n_t^y, n_t^s, m_t^{s,s}, m_t^{y,y}, m_t^{y,s}, m_t^{y,y}, m_t^{s,y}, m_t^{s,y}; p_t^s, p_t^y, p_t^l, r_t, R_t\}_{t=0}^\infty \), and prices \( \{w_t, t_t^k, p_t^l, p_t^s, r_t, R_t\}_{t=0}^\infty \) that satisfy the following:

1. Consumers’ optimization problem,

2. Profit maximization in the manufacturing and construction sector,

3. Clearing of markets:

   (a) **Labor markets** (\( w_t \))

   \[ n_t = n_t^y + n_t^s, \quad \forall t, \]

   (b) **Land markets** (\( p_t^l \))

   \[ l_t = l_{t-1} = \bar{l}, \quad \forall t, \]

   (c) **Market rental capital** (\( r_t^k \))

   \[ r_t^k = A_t^y F_1(k_t, n_t^y, m_t^{y,y}, m_t^{y,s}), \quad \forall t, \]

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(d) Goods markets \((p^*_t = 1)\)
\[ c_t + x^{k}_t + m^{y,y}_t + m^{s,s}_t = A^y_t F(k_t, n^y_t, m^{y,y}_t, m^{s,s}_t), \quad \forall t, \]

(e) Construction of structures \((p^*_s)\)
\[ x^{sh}_t + x^{s,y}_t + m^{s,y}_t + m^{s,s}_t = A^s_t G(n^s_t, m^{s,s}_t), \quad \forall t. \]

We have assumed complete markets from the outset; hence this equilibrium is efficient. The details of the optimization problem solved are discussed in Appendix.

5 Quantitative Analysis

5.1 Parameterization

The quantitative evaluation of the model requires specifying parameter values and functional forms. The choice of functional forms is relatively general. The utility function is consistent with unitary income elasticity,
\[ u(c, h, N) = \left[ \frac{\eta c^{-\rho} + (1 - \eta)h^{-\rho}}{1 - \sigma} \right]^{1-\sigma}, \]
where the parameter \(\rho\) pins down the elasticity of substitution between consumption, \(c\), and housing services, \(h\); the parameter \(\sigma\) represents the intertemporal elasticity of substitution; and \(\eta\) represents the relative importance of consumption. The function capturing the utility from leisure is logarithmic, as is standard in the real business cycle literature with a representative agent,\(^9\)
\[ v(1 - n) = \rho \log(1 - n). \]

The production of housing services is also given by a Cobb-Douglas specification,
\[ h = H(s, l) = z_h (s)^{\epsilon} (l)^{1-\epsilon}, \]
where \(z_h\) represents a transformation factor between stock and the flow and \(\epsilon\) represents the share between structures and land. The production of consumption goods uses Cobb-Douglas technology
\[ F(k, n^y, m^{y,y}, m^{s,y}) = A^y(k)^{\alpha_1} (n^y)^{\alpha_2} (m^{y,y})^{\alpha_3} (m^{s,y})^{1-\alpha_1-\alpha_2-\alpha_3}, \]
where \(\alpha_i\) represents the share in production for input \(i\) and has constant returns to scale. Notice that the specification allows for substitutability between intermediate goods. The technology used in the construction sector is a CES with diminishing returns to scale \(\sum_i \vartheta_i < \)

\(^9\)This specification implies a Frisch elasticity of labor equal to 2. Keane and Rogerson (2012) argue that this elasticity can be reconciled with lower elasticity estimates at the micro level.
1,
\[ G(n^s, m^{s,s}, m^{y,s}) = A^s \left[ \gamma_2 (n^s)^{-\gamma_4} + (1 - \gamma_2) \left( (m^{s,s})^{\gamma_3} (m^{y,s})^{1-\gamma_3} \right)^{-1/\gamma_4} \right]. \]

The model parameters are set to match long-run averages of their data counterparts between 1952 and 2000. The implied parameter values are relatively robust to choice of the sample period; however, during the housing boom some of the ratios and long-run averages departed significantly from the historical trend. The model targets need to be adjusted to be consistent with the data counterparts.

The time unit is a year, as input-output tables are yearly at best. The discount factor is \( \beta = 0.96 \). The depreciation rates of residential structures and nonresidential capital are \( \delta^s = 0.015 \) and \( \delta^y = 0.115 \), respectively. The weight on leisure, \( \rho = 0.33 \), is such that total hours worked equal one-third of the time endowment in steady state. The preference parameters are set to match consumption-to-output and housing-to-output ratios. The parameters of the production functions are set to satisfy the following:

1. The ratio of gross output in the two sectors, \( Y^s/Y^y = 0.08 \)
2. Observed labor share in the construction sector, \( = 0.7 \)
3. Observed labor share in the manufacturing sector, \( = 0.65 \)
4. The ratio of consumption to manufacturing gross output, \( = 0.35 \)
5. Observed shares of intermediates in gross output of own sector (\( M^{sy} \) and \( M^{ys} \)), \( = 0.4, 0.007 \)
6. Time allocated to market activities, \( n^y + n^s = 1/3 \)
7. The ratio of employment in the two sectors, \( n^y/n^s = 16 \)

The values of the parameters not mentioned above are displayed in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( \alpha_1 )</th>
<th>( \alpha_2 )</th>
<th>( \alpha_3 )</th>
<th>( \gamma_1 )</th>
<th>( \gamma_2 )</th>
<th>( \gamma_3 )</th>
<th>( \gamma_4 )</th>
<th>( A^y )</th>
<th>( A^s )</th>
<th>( z_h )</th>
<th>( \epsilon )</th>
<th>( \eta )</th>
<th>( \sigma )</th>
<th>( \rho )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
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<td>0.5</td>
<td>0.035</td>
<td>0.62</td>
<td>0.4</td>
<td>0.04</td>
<td>1.5</td>
<td>2.4</td>
<td>1.74</td>
<td>0.175</td>
<td>0.28</td>
<td>0.435</td>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>

The intratemporal elasticity of substitution between consumption and housing services is determined by the parameter \( \varepsilon_{ch} = 1/(1 + \rho) \). Quantitatively, the value of \( \rho \) is an important determinant of the spillover effects from the housing services demand into goods. If consumption services are close substitutes, a decline in the demand for housing services can generate an increase in goods consumption, whereas if these are close complements a decline in housing demand translates to the goods sector. Recent papers in an extensive literature estimate this elasticity to be less than 1. For example, Flavin and Nakagawa (2008) use a model of housing demand and estimate an elasticity less than 0.2. Other papers (i.e.,
Song, 2010, and Landvoight, 2011) use alternative model specifications and estimate values for the elasticity to be less than 1. The simulations consider different ranges of elasticities $\varepsilon_{ch} \in \{0.17, 0.25\}$.

The other key parameter is $\theta_t$, which is used here to affect the demand for housing. We perform the following exercise. Starting in 2001 the value $\theta_t$ grows to match employment patterns in the construction sector. Each new increment is a surprise (i.e., people expect $\theta_t$ not to change after that). The housing boom lasts until 2007, the housing bust lasts 3 years, and thereafter $\theta_t$ remains constant forever. Figure 7 summarizes the implied path for employment in the construction sector in the model and the data. For the sequence of demand shifters, the model can reproduce the observed pattern quite well.

Figure 7: Employment Construction Sector (Model and Data)

5.2 Role of Residential Investment in Growth and Employment

The key in this exercise is the behavior of other macroeconomic quantities (output, total employment, and intermediate production) during the transition path. They are endogenously determined from the initial steady state. The baseline case considers a boom and bust in the construction sector with sectoral adjustments in the interdependent sectors, $m = \{m_t^{y, s}, m_t^{y, y}, m_t^{y, y}, m_t^{s, y}\}_{t=0}^{\infty}$. The implications of the boom and bust in construction for output and total employment are summarized in Figure 8.

**Boom in housing demand and construction.** Upon the arrival of the initial shift in housing demand, the shock of residential structures is fixed and cannot be adjusted in the short run. In principle, the higher demand for housing driven by higher values of $\theta$ may lower, in relative terms, the marginal utility of consumption. Because of the complementarity, goods consumption remains essentially unchanged during the initial period of the boom. Demand is higher, and thus to clear markets, the price of the construction sector must increase. In the following periods, the quantity of residential structures can be adjusted. The latter increases the value of the marginal product of inputs used in production. Higher demand for inputs ultimately means the construction sector expands. The construction sector requires, among its inputs, intermediate goods purchased from the nonhousing sector. This increased demand from output of the nonhousing sector results, in equilibrium, in higher investment for this sector with higher capital and employment levels and ultimately more output. The
expansion of the construction sector generates growth in the consumption goods sector and increases total employment in the economy. All of these forces operate as long as preferences for housing continue to increase.

**Figure 8: The Impact of Construction**

**GDP**

<table>
<thead>
<tr>
<th>Year</th>
<th>GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>0.98</td>
</tr>
<tr>
<td>2002</td>
<td>0.99</td>
</tr>
<tr>
<td>2004</td>
<td>1.00</td>
</tr>
<tr>
<td>2006</td>
<td>1.01</td>
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<tr>
<td>2008</td>
<td>1.02</td>
</tr>
<tr>
<td>2010</td>
<td>1.03</td>
</tr>
<tr>
<td>2012</td>
<td>1.04</td>
</tr>
<tr>
<td>2014</td>
<td>1.04</td>
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<td>2016</td>
<td>1.04</td>
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<tr>
<td>2018</td>
<td>1.04</td>
</tr>
<tr>
<td>2020</td>
<td>1.04</td>
</tr>
</tbody>
</table>

**Employment**

<table>
<thead>
<tr>
<th>Year</th>
<th>Employment</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>0.98</td>
</tr>
<tr>
<td>2002</td>
<td>0.99</td>
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<td>2004</td>
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<td>2016</td>
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<tr>
<td>2018</td>
<td>1.04</td>
</tr>
<tr>
<td>2020</td>
<td>1.04</td>
</tr>
</tbody>
</table>

**Bust in housing demand and construction.** Similar forces operate once the crash in housing demand starts. The decline in the stock of residential structures is bounded by the relatively low rate of depreciation, which forces a strong decline in the price of the construction sector. Employment in the construction sector falls dramatically, together with the demand for all other inputs of this sector. The input-output structure of the model once again lowers the demand for the consumption good sector as intermediate demand from the construction sector falls. In the short run, the decline in the demand for housing generates a very small and short-lived increase in nonhousing consumption. This effect would be missing in a model including negative wealth effects from a decline in house prices.

The magnitude of quantitative implications depends on the assigned value for the elasticity between consumption and housing services. With perfect complementarity, both goods should comove perfectly, whereas with both goods close to perfect substitutes, the decline in the demand for housing could even imply a boom in the goods sector. In the baseline calibration, the temporary consumption increase is not sufficient to compensate for the decline in other key macroeconomic aggregates; as a result, GDP falls 3.3 percent and total employment declines 4 percent. The asymmetry between the boom and the bust depends on the sequence of demand shifters but it also depends on the number of periods the irreversibility constraint binds. This amplifies the asymmetric response between the boom and the bust episode. When this effect is quantitatively important the decline of output is 3.3 percent and the employment decline is 3.8 percent.

The housing boom in the model generates significant movements in total employment. Recall that the sequence of demand parameters, \( \theta_t \), is calculated to replicate the evolution of employment in the construction sector. Fluctuations in construction employment were important during the Great Recession. Thus, the model can capture their timing and mag-
plitude relatively well. In relative terms, the boom drew employment from the goods sector into housing, and the bust sent workers outside construction and out of the labor force. The long-run value of $\theta$ is determined to replicate the decline in employment in the construction sector; as a result, the long-run level of employment declines relative to the initial steady state. The temporary decline in employment occurs regardless of the level of the terminal conditions.\textsuperscript{10} Because of the slow adjustment of the stock of structures combined with the irreversibility constraint, there is a significant adjustment and the construction sector remains inactive for a long period.

To compare the model implications for different elasticity values, it is necessary to re-calculate in each case a sequence of demand for housing $\{\theta_t\}$ that matches the dynamics of employment in the construction sector. The qualitative implications in both cases are the same, but quantitatively the model with higher complementarity has a larger effect during the housing boom and the bust. With a lower degree of complementarity GDP falls 2 percent instead of 3.3 percent and total employment declines by 2.4 percent instead of 4 percent.

The model also has some implications for the path of other aggregates. As shown in Figure 9, the dynamics are quite similar for the different elasticity parameters. The most significant changes are the evolution of house prices and intermediates produced by the construction sector. The change in housing demand generates modest movements in house prices: a 10 percent increase during the housing boom and a 15 percent decline during the bust. Movements in house prices have effects in the production of goods from the construction sector. Construction produces more goods as the price increases, but the goods

\textsuperscript{10} An alternative experiment would be one in which housing demand remains low for a number of periods before returning to the initial steady-state level. We have computed the experiment, and the quantitative implications are not very different. However, the long-run level of employment is higher.
sector responds by demanding less of the construction sector and using more capital.

Figure 9: Summary of Key Aggregates

One should not expect that dynamics of this economy that responds to a single shifter to match the data counterparts. For example, the decline in housing demand generates a small increase in consumption goods not observed in the data. Adding dimensions that affect the household budget constraint should improve the performance of the model, but it would also make it more complicated to interpret the contribution of interlinkages.

5.3 The Role of Interlinkages

The interlinkages can be an important driver of aggregate output and total employment during the housing boom and bust. To isolate the effects of the interlinkages from those derived purely from consumers’ demand for housing, we consider two alternative specifications. The first alternative considers the same model calibration and controls for the marginal effects of the production of intermediate goods to the aggregate. This is done by restricting the sectoral demand of intermediates to be fixed and consistent with the quantities produced in the initial steady state in 1998 ($m_{t}^{s,s} = m_{0}^{s,s}, m_{t}^{y,s} = m_{0}^{y,s}, m_{t}^{y,y} = m_{0}^{y,y}, m_{t}^{s,y} = m_{0}^{s,y}$). This case is referred to as “no interlinkages.” A second alternative to identify the impact of housing demand on employment and activity is to compare the economy with interlinkages with an
economy that measures everything at the value-added level. In the value-added economy, the relevant technologies in the absence of interlinkages are

\[ c_t + x^k_t = A^y_t F(k_t, s^y_t, n^y_t), \]

and

\[ x^{sh}_t + x^{sy}_t = A^s_t G(n^*_t). \]

In both specifications the quantitative experiment is the same. The housing demand shifter is adjusted to generate movements in construction employment consistent with the data. Figure 10 compares the key macroeconomic aggregates in the case of interlinkages, no interlinkages, and value added in the case of \( \rho = 5 \).

**Figure 10: The Impact of Construction: Role of Interlinkages**

The role of the input-output linkages is clear when we compare the economy subject to the same sequence \( \{\theta_t\} \) but with no adjustment of intermediate goods. Now, the intermediates are fixed to the initial steady-state levels. Both sectors are committed to produce the same amount of intermediates every period. During the housing boom, the only way to produce more homes is to use more capital and labor. Since the intermediates cannot adjust, the prices adjust more, and rents and the price of construction are more volatile (Figure 11). Qualitatively speaking, equilibrium dynamics in this version of the model where intermediates are constrained to be constant are quite similar to those of the baseline experiments. However, the quantitative implications are very different. Since intermediates are constant, the marginal product of labor in the construction sector does not increase as much as in the baseline experiment, and employment also does not increase as much. The construction sector expands during the boom, but in this model we do not allow for direct intersectoral links; in particular, the demand for intermediates of the construction sector from the non-housing sector is not allowed to change. As a result, the nonhousing sector barely changes in

\[ \text{See the Appendix for model details.} \]
this experiment (all movements are less than one-half of 1 percent). GDP and employment changes are an order of magnitude smaller than in the economy with intersectoral links. The input-output linkages operate as total factor productivity changes, amplifying the boom and the bust. In the value-added model, the change in the demand for housing also generates a very small boom and bust in output and employment. All change is driven by the demand side of the model; the lack of interlinkages fails to amplify the effects in the construction sector to other sectors in the economy. In these two cases, the irreversibility constraints have no effects on the dynamics during the housing bust.

Figure 11: Other Variables: Role of Interlinkages

An indirect implication of the model is the dynamics of rental prices computed from the marginal rate of substitution between housing and consumption. The models that generate very small movements in aggregate quantities also require significant movements in the rental price. The price increase is necessary to restrain consumers despite the increase in their desire to consume more housing. When the levels of interlinkages are fixed or nonexistent, the lack of adjustment behaves as a negative productivity shifter in each sector. In the model with interlinkages, the interconnection operates as a positive productivity factor on capital and employment. The positive response in quantities reduces the movement in the rental price.

The alternative specifications illustrate an important point. The presence of interlinkages is necessary to generate large aggregate changes from fluctuations in construction. Importantly, notice that both alternative models generate little change in terms of GDP
and employment even though the complementarity between consumption and housing is not modified. This implies that this complementarity is not sufficient to obtain the propagation of shocks to the construction sector highlighted here.

5.4 Quantitative Implications of Alternative Models

The different model specifications point to a similar conclusion: The effect of the construction sector is significant despite its relatively small share in terms of employment and value added. Table 3 presents a summary of all results presented above. The table shows information about the share of the change generated by the different models that occurs during the expansion and recession. Because changes in the model are due only to changes in the demand of housing services, we refer to those shares as the share of changes accounted for by the construction sector. The variables considered are employment and GDP. Notice that when all sectors are growing at the same rate, the contribution of each sector must be equal to its share. For instance, if construction employment represents 5 percent of aggregate employment, we should expect this sector to contribute 5 percent to the expansion of the economy. This is not necessarily the case when sectors are growing at different rates.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Expansion 2002-07</th>
<th>Recession 2008-09</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Employment (%)</td>
<td>GDP (%)</td>
</tr>
<tr>
<td>Lower complementarity ($\rho = 3$)</td>
<td>29.2</td>
<td>8.3</td>
</tr>
<tr>
<td>Higher complementarity ($\rho = 5$)</td>
<td>61.1</td>
<td>14.9</td>
</tr>
<tr>
<td>Value added</td>
<td>15.2</td>
<td>1.9</td>
</tr>
<tr>
<td>Fixed intermediates</td>
<td>14.7</td>
<td>1.5</td>
</tr>
</tbody>
</table>

The left side of Table 3 considers the role of the construction sector in the expansion between 2002 and 2007. Regardless of the complementarity between housing and consumption goods, our use of a general equilibrium model with interlinkages reveals that the construction sector accounts for a very significant share of the growth in employment: between 29 percent and 61 percent. The contribution of construction to GDP was also larger than its share—between 8 percent and 15 percent— but much smaller than for employment. Recall that this period was usually referred to as the “jobless recovery,” in which most of the growth in employment was created by construction.

The effect of construction was arguably even more important during the Great Recession. The decline in employment generated in the models with interlinkages is between 28 percent and 43 percent of the actual decline during the recession. In the case of GDP, the models generate between 43 percent and 60 percent of the observed changes during the recession.
Thus, construction accounts for very significant shares of the expansion and recession over the past decade.

Interlinkages are clearly important in the model. Without these channels (referred to as “fixed intermediates” or “value added”), changes in the construction sector have much smaller effects on aggregate output and total employment, even when consumption goods and housing services are complements. Recall that the value reported in this case corresponds to $\rho = 5$.

6 Interlinkages and Business Cycle Accounting

An alternative methodology to identify the sources of the recent recession is business cycle accounting based on Chari, Kehoe, and McGrattan (2007). Recent work, including Arellano, Bai, and Kehoe (2010), and Ohanian and Raffo (2012), documents that the Great Recession can be accounted for mostly by a worsening of labor market distortions. Both studies find that the labor wedge worsens by about 12 percent during the 2009 recession. Different explanations have been proposed to rationalize the increase in distortions in the labor market. For instance, Arellano, Bai, and Kehoe (2010) propose a model of imperfect financial markets and firm-level volatility. Their model captures about half of the worsening in the labor wedge.

This wedge can be computed with data on employment, consumption, and wages generated with any model. In particular, the wedge is defined as

$$X_t = -\frac{U_{N_t}}{U_{C_t}}/w_t,$$

where $U_{N_t}$ is the marginal disutility measured at the aggregate level of employment, $U_{C_t}$ is the marginal utility of consumption measured at the aggregate level of consumption, and $w_t$ is the economy wage. Assuming wages are flexible and considering an aggregate Cobb-Douglass production function with capital share $\alpha$, the wage can be replaced with

$$w_t = \frac{Y_t}{N} (1 - \alpha).$$

Furthermore, using a log utility function for aggregate consumption and the following function for the disutility of aggregate employment,

$$U(N) = B \frac{N^{1+v}}{1 + v},$$

the wedge can be written as

$$\Gamma_t = -\frac{B}{(1 - \alpha)} \frac{C_t}{Y_t} N_t^{1+v}.$$

Notice that the parameters $B$ and $\alpha$ are not important to understand fluctuations in the labor wedge; only time series of consumption, output, and employment at the aggregate
level and a value for $v$ are required. We consider three values of $v = \{0.5, 1, 2\}$ and compute the labor wedge implied by our model using simulated data for consumption, output, and employment. Since our model has multiple sectors, several adjustments in the data are necessary. Consumption of goods and housing services are aggregated using relative prices $C_t = c_t + R_t h_t$. Aggregate output is $Y_t = C_t + X^k_t$ and total employment is $N_t = n^y_t + n^s_t$.

In the context of the model, any action in terms of implied distortions must be derived from the input-output structure and changes in relative prices. Figure 12 displays the changes in the labor wedge for our benchmark simulation. The behavior is consistent with the data. The labor wedge worsens during the recession and does not recover quickly. For the case computed with $v = 1$, which is consistent with the value used by Ohanian and Raffo (2012), the labor wedge worsens 7.4 percent; this is about 62 percent of the total labor wedge change during this period.

![Figure 12: Labor Wedge](image)

Notice that our computation of the labor wedge assumes that wages are perfectly flexible. If this condition does not hold, the labor wedge has another component, referred to as the “firm-side” labor wedge in Arellano, Bai, and Kehoe (2010).\textsuperscript{12} This wedge is basically the difference between the marginal product of labor and the wage. These authors refer to the other component of the labor wedge as the “consumer-side” labor wage, which is basically our $\Gamma_t$. Arellano, Bai, and Kehoe (2010) find that (i) the firm-side labor wedge has been fairly flat since 2006 and (ii) a worsening of the consumer-side labor wedge accounts for most of the Great Recession. Recall that there are no frictions in our model, so wages equal the marginal product of labor in every period. Thus, not only the behavior of the labor wedge during the Great Recession but also its decomposition is consistent in our model and the data.

\textsuperscript{12}They follow Galí, Gertler, and López-Salido (2007) in this decomposition.
7 Conclusions

This paper analyzes the contribution of the construction sector to U.S. economic growth, particularly during the Great Recession. Historically, the construction sector has been relatively small in terms of employment and contribution to GDP, but it is highly interconnected with other sectors in the economy. Our empirical analysis reveals these sectoral interlinkages propagate changes in housing demand, amplifying the effect in the economy. A simple accounting input-output framework reveals that construction accounts for 52 percent of the decline in employment and 35 percent of the decline in output.

The importance of the sectoral interlinkages is illustrated using a simple static multisector model. In the model, changes in housing demand have a much larger effect on aggregate activity when the sectors are interconnected. The presence of irreversibility constraints introduces an asymmetry between the expansion and the recession in the dynamic model. The simulation exercise is calibrated to reproduce the boom-bust dynamics of construction employment in the 2000-10 period. In the model, during the housing boom all sectors expand and contribute 2 percent and 2.5 percent to the growth of output and employment between, respectively. During the housing bust, the irreversibility constraint binds, amplifying the asymmetric response; the declines in output and employment are 3.3 percent and 3.8 percent, respectively. The asymmetric effect is not as large but still significant with a lower degree of complementarity. These numbers can be used to calculate the contribution of construction in the observed data. The model suggests that during the expansion (2002-07), the construction sector accounts for a significant share of the growth in employment (between 29 percent and 61 percent) and GDP (between 8 percent and 15 percent). The construction sector’s contribution was more important during the Great Recession (2008-09) when declines in employment and GDP ranged between 28 percent and 43 percent and 43 percent and 60 percent, respectively.

The presence of intersectoral linkages substantially amplifies the impact of the changes in housing demand. In the model specifications without this mechanism, changes in housing demand consistent with the dynamics in construction employment have only a small effect on macroeconomic quantities. This is true even when the complementarity between consumption goods and housing services is high. A direct implication of this result is the presence of interlinkages is necessary to generate large aggregate changes from changes in construction, and the degree of complementarity is not sufficient to obtain the propagation of adjustments in housing demand to the construction sector highlighted here.

We use this analytical framework to interpret the behavior of the U.S. economy between 2007 and 2011. Since in our model the equilibrium is efficient, the behavior of output is also the behavior of potential output. Taking into account that both output and potential output were affected during the Great Recession, we perform a business cycle accounting exercise on simulated data from the model. Despite the lack of frictions and distortions in the model, the sectoral linkages and movements of relative prices across sectors attribute the recession to a worsening in the labor wedge. The magnitude generated by the model accounts for 62
percent of the total change observed in the data.

A direct policy implication of our findings is that the output gap could be lower than the historical estimates. The historical anomalies in the events during the past five years can be accounted for by the equally anomalous evolution of housing demand in the previous years. As far as policy is concerned, the basic implication of our research is simple: Estimations of output gaps using pre-2007 trends may lead to inaccurate policy actions.

This model abstracts from sizable income and wealth effects due to deleveraging and mortgages. As a result, all short-run dynamics are driven entirely by substitution effects. The interaction between the financial factors used in Garriga, Manuelli, and Peralta-Alva (2012ab) with the production structure of the paper should magnify the importance of the sectoral interlinkages. The inclusion of wealth effects and frictions from housing finance should be a natural extension.

References


8 Appendix

8.1 Different Model Specifications with and without Interlinkages

In the quantitative analysis it is important to disentangle the role of interlinkages. It is always challenging to compare different models, but the quantitative analysis suggests similar results from the various alternatives. The first alternative considers an economy calibrated to the same initial steady state (parameters and targets) and compares the economy with
interlinked production with an economy that fixed the size of intermediates to the initial steady-state levels. The second alternative compares the value-added economy with the economy with interlinkages. Both economies are calibrated to the same target values for the baseline year, but the underlying parameters are different.

There is an optimization problem that solves for the equilibrium in each case. Those cases are presented here with a slightly more general assumption: The production function $F$ also uses structures, referred to as $s^y_t$. The structures used for the production of housing services are now referred to as $s^h_t$ instead of just $s_t$.

In the baseline case with interlinkages, the social planner problem chooses a sequence of quantities $\{c_t, x^k_t, x^{sh}_t, x^{sy}_t, l_t, n^y_t, n^s_t, m^{s,s}_t, m^{y,y}_t, m^{y,y}_t, m^{s,y}_t\}_{t=0}^\infty$ to maximize

$$\max \sum_{t=0}^\infty \beta^t [u(c_t, \theta_t, h_t) + \gamma v(1 - n^y_t - n^s_t)],$$

subject to

$$c_t + x^k_t + m^{y,y}_t + m^{y,y}_t = A^y_t F(k_t, s^y_t, n^y_t, m^{y,y}_t), \quad \forall t,$$

$$x^{sh}_t + x^{sy}_t + m^{s,s}_t + m^{s,y}_t = A^y_t G(n^s_t, m^s_t(m^{s,s}_t, m^{s,y}_t)), \quad \forall t,$$

$$x^k_t = k_{t+1} - (1 - \delta_k)k_t \geq 0, \quad \forall t,$$

$$x^{sh}_t = s^{h}_t - (1 - \delta_{sh})s^h_t \geq 0, \quad \forall t,$$

$$x^{sy}_t = s^y_t - (1 - \delta_{sh})s^y_t \geq 0, \quad \forall t,$$

$$h_t = H(s^h_t, \bar{t}), \quad \forall t,$$

$$s_0, l_0, k_0 \geq 0.$$

In the model with no interlinkages, the production of intermediate goods is fixed to the initial steady-state levels before the housing boom. In this case, the social planner is forced to produce the same quantity of intermediates each period ($m^{s,s}_t = m_0^{s,s}, m^{y,y}_t = m_0^{y,y}$). To satisfy the production of the intermediate goods, the social planner can choose a constrained vector of quantities $\{c_t, x^k_t, x^{sh}_t, x^{sy}_t, l_t, n^y_t, n^s_t\}_{t=0}^\infty$ to maximize

$$\max \sum_{t=0}^\infty \beta^t [u(c_t, \theta_t, h_t) + \gamma v(1 - n^y_t - n^s_t)],$$

subject to

$$c_t + x^k_t = A^y_t F(k_t, s^y_t, n^y_t, m^{y,y}_t) - (m^{y,y}_0 + m^{y,y}_0), \quad \forall t,$$

$$x^{sh}_t + x^{sy}_t = A^y_t G(n^s_t, m^s_t(m^{s,s}_0, m^{s,y}_0)) - (m^{s,s}_0 + m^{s,y}_0), \quad \forall t,$$

$$x^k_t = k_{t+1} - (1 - \delta_k)k_t \geq 0, \quad \forall t,$$

$$x^{sh}_t = s^{h}_t - (1 - \delta_{sh})s^h_t \geq 0, \quad \forall t,$$

$$x^{sy}_t = s^y_t - (1 - \delta_{sh})s^y_t \geq 0, \quad \forall t,$$

$$h_t = H(s^h_t, \bar{t}), \quad \forall t,$$

$$s_0, l_0, k_0, m_0^{s,s}, m_0^{y,y}, m_0^{y,y}, m_0^{s,y} \geq 0.$$

The last case considers the value-added economy where the intermediate goods have been
completely eliminated. The social planner can choose a constrained vector of quantities \( \{c_t, x_{kt}^k, x_{ht}^s, x_{yt}^y, l_t, n_t^y, n_t^s\}_{t=0}^\infty \) to maximize

\[
\max \sum_{t=0}^\infty \beta^t [u(c_t, \theta_t h_t) + \gamma v(1 - n_t^y - n_t^s)];
\]

subject to

\[
\begin{align*}
  c_t + x_{kt}^k &= A_t^k f(k_t, s_t^y, n_t^y), & \forall t, \\
  x_{ht}^s + x_{yt}^y &= A_t^s G(n_t^s), & \forall t, \\
  x_{kt}^k &= k_{t+1} - (1 - \delta_k)k_t \geq 0, & \forall t, \\
  x_{ht}^s &= s_{t+1}^h - (1 - \delta_{sh})s_t^h \geq 0, & \forall t, \\
  x_{yt}^y &= s_{t+1}^u - (1 - \delta_{sy})s_t^y \geq 0, & \forall t, \\
  h_t &= H(s_t^h, \bar{I}), & \forall t, \\
  s_0, l_0, k_0 &\geq 0,
\end{align*}
\]

### 8.2 Calibration of Interlinkages

Interlinkages are calibrated using input-output data. In particular, the information shown in Table A1 is used to calibrate the parameters in the production function of consumption goods and residential structures. In particular, the tables are constructed from the 2010 BEA’s Use input-output table. The Use table shows the uses of commodities by intermediate and final users; rows present the commodities or products, and columns display the industries and final users that use them. The sum of the entries in a row is the output of that commodity. The columns show the products consumed by each industry and the three components of “value added”—compensation of employees, taxes on production and imports less subsidies, and gross operating surplus. Value added is the difference between an industry’s output and the cost of its intermediate inputs, and total value added is equal to GDP.

Table A1 displays input-output values (which are originally in millions of dollars) as a fraction of the industries’ outputs. Construction receives most of its inputs from other industries (48.3 percent of its gross output) and less than 1 percent from itself. This is also true for other industries since they receive most of their inputs from themselves (43.0 percent...
of their total gross output).

<table>
<thead>
<tr>
<th>Commodities/Industries</th>
<th>Construction</th>
<th>Other Industries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construction</td>
<td>0.0009</td>
<td>0.0058</td>
</tr>
<tr>
<td>Other industries</td>
<td>0.4828</td>
<td>0.4301</td>
</tr>
<tr>
<td>Compensation of employees</td>
<td>0.3625</td>
<td>0.2802</td>
</tr>
<tr>
<td>Taxes on production and imports, less</td>
<td>0.0072</td>
<td>0.0471</td>
</tr>
<tr>
<td>subsidies</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gross operating surplus</td>
<td>0.1466</td>
<td>0.2368</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1.0000</strong></td>
<td><strong>1.0000</strong></td>
</tr>
</tbody>
</table>

Source: Use input-output table (BEA).

### 8.3 The Case of Complementarity between Consumption and Housing

The generalized version of the simple model illustrates that in the case of complementarity between consumption and housing, a decline in the demand for housing can also reduce the demand for consumption goods. Consider the extreme case where both goods are perfect complements, \( u(c, h, n) = \min\{c, \theta h\} - an \). In this case, a decline in the demand for housing \( \theta' < \theta \) also reduces the demand for consumption goods. In the other extreme case, both goods “consumption and housing” are perfect substitutes, \( u(c, h, n) = c + \theta h - an \), so a decline in the demand for housing generates an increase in the demand for the other sector. This model analytically illustrates how the degree of complementarity between goods and the interlinkages is important in understanding the impact on total output and employment.

Consider a more general specification for the consumer preferences. The index for consumption goods and housing is CES, where the parameter \( \rho \) represents the elasticity between consumption goods and housing, and the supply of labor is convex where \( \gamma > 0 \) represents the curvature. In the case of interdependence, the social planner solves

\[
\max_{c, h, n, n^s, n^y} \left\{ \left( c^\rho + \theta h^\rho \right)^{\frac{1}{\rho}} - a \frac{n^{1+\gamma}}{1+\gamma} \right\},
\]

s.t.

\[
c + m^y = A^y \min\{n^y, \frac{m^y}{e^y} \},
\]

\[
h + m^s = A^s \min\{n^s, \frac{m^s}{e^s} \},
\]

\[
(n^s + n^y) = n.
\]

Rearranging terms yields
\[
\max_{c,h,n,n^*} \{[c^\rho + \theta h^\rho]^{\frac{1}{\rho}} - a \frac{n^{1+\gamma}}{1 + \gamma} \}, \\
s.t. \quad c = A^y n - n^s (A^y + \varepsilon^s), \\
\quad h = (A^s + \varepsilon^y) n^s - \varepsilon^y n.
\]

The first-order conditions are
\[
\begin{align*}
\text{c} : \quad & [c^\rho + \theta h^\rho]^{\frac{1}{\rho}-1} c^{\rho-1} = \lambda_1, \\
\text{h} : \quad & [c^\rho + \theta h^\rho]^{\frac{1}{\rho}-1} \theta h^{\rho-1} = \lambda_2, \\
\text{n} : \quad & -a n^\gamma + \lambda_1 A^y - \lambda_2 \varepsilon^y = 0, \\
\text{n}^s : \quad & -\lambda_1 (A^y + \varepsilon^s) + \lambda_2 (A^s + \varepsilon^y) = 0.
\end{align*}
\]

To derive the optimal employment decision, combine, the first-order conditions
\[
\frac{\lambda_1}{\lambda_2} = \frac{(A^s + \varepsilon^y)}{(A^y + \varepsilon^s)},
\]
and
\[
\frac{(A^s + \varepsilon^y)}{(A^y + \varepsilon^s)} = \frac{1}{\theta} \left( \frac{h}{c} \right)^{1-\rho},
\]
or
\[
\left[ \frac{\theta (A^s + \varepsilon^y)}{(A^y + \varepsilon^s)} \right]^{\frac{1}{1-\rho}} = \frac{A^y n - n^s (A^y + \varepsilon^s)}{(A^s + \varepsilon^y) n^s - \varepsilon^y n}.
\]

The optimal allocation of labor across sectors is
\[
n^s = \frac{z A^y + \varepsilon^y}{(A^s + \varepsilon^y) + z (A^y + \varepsilon^s)} n,
\]
and
\[
n^y = \frac{A^s + z \varepsilon^s}{(A^s + \varepsilon^y) + z (A^y + \varepsilon^s)} n.
\]

The elasticity between consumption and housing affects the term \( z \). For \( \rho = 0 \), the model implies the labor demand derived in Section 3 is a special case. The optimal level of employment in each sector is a linear function of the total employment \( n^s = \alpha n \) and \( n^y = (1 - \alpha) n \).

The linearity is useful to define output in the consumption and housing sectors as expressions of \( n \):
\[
c = [(1 - \alpha) A^y - \alpha \varepsilon^s] n,
\]
and
\[
h = [\alpha A^y + (1 - \alpha) \varepsilon^y] n.
\]
Now the Lagrange multipliers are a function of the total level of employment \(\lambda_1(n)\) and \(\lambda_2(n)\). Then, use this optimality condition to determine the optimal level of employment:

\[
n = \left[ \frac{1}{\alpha} \left( [(1 - \alpha)A^y - \alpha \varepsilon^s]^{\rho} + \theta [\alpha A^y + (1 - \alpha) \varepsilon^y]^{\rho} \right)^{-1} \left( \frac{A^y}{[(1 - \alpha)A^y - \alpha \varepsilon^s]^{1-\rho}} - \frac{\theta \varepsilon^y}{[\alpha A^y + (1 - \alpha) \varepsilon^y]^{1-\rho}} \right) \right]^{\frac{1}{\rho}}.
\]

With complementarity in preferences, total employment depends not only on the demand for the construction sector, but also on the indirect effects on the total demand for goods.