Grown-up Business Cycles*

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Abstract

We document two striking facts about US firm dynamics and interpret their significance for aggregate employment dynamics. The first observation is the steady decline in the firm entry rate over the last thirty years, and the second is the gradual shift of employment from younger to older firms over the same period. Using data from the Census Bureau Business Dynamics Statistics (BDS) and a simple model of firm dynamics, we show that shift of employment towards older firms is entirely a cumulative effect of the decline in firm entry; there has been little change in firm survival and growth rates conditional on entry. These trends have a substantial effect on aggregate employment dynamics: 1. the decline in job creation from firm entry reduces the trend growth rate of employment; 2. the gradual aging of firms acts to decrease the cyclical sensitivity of employment. Put together these two forces imply a decline in the cyclical sensitivity of employment growth during expansions while their effect is offsetting during contractions. Consequently, employment recovers slower relative to output in more recent expansions, as consistent with the data. Finally, we consider the slow recovery from the Great Recession and show that employment recovery to the pre-recession peak would have taken place a full two years ahead of the current recovery if the startup rate and age distribution of employment were the same as they were in the late 1980s.

1 Introduction

There has been a dramatic decline in the firm startup rate in the US for the last thirty years. Figure 1a shows as a solid line the share of age 0 employer firms (what we refer to as startups) as a fraction of the overall stock of employer firms, or startup rate. Since the early 1980s the startup rate has

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Figure 1: Firm and employment share of startups and mature firms

Note: US Census Bureau Business Dynamics Statistics. Left panel: Number of age 0 employer firms as fraction of number of employer firms of all ages (left axis) and total employment at age 0 firms as fraction of total employment at firms of all ages. Series begins in 1977 because age 0 employers are not left censored. US Census Bureau Business Dynamics Statistics. Right panel: Number of age 11+ employer firms as fraction of number of employer firms of all ages and total employment at age 11+ firms as fraction of total employment at firms of all ages. Series begins in 1987 because firms aged 11+ are left censored from 1977 to 1986.

dropped from an average of about 13 percent to a recent level near 8 percent. Since the average startup employment has remained roughly constant over this period, while the overall average firm size has slightly increased, the employment share of startup firms has declined even faster. Plotted as a broken line, the startup share of employment has fallen by almost half over this period from 4 percent to just above 2 percent. Over the same period, much older incumbent firms have become more prominent and gained a larger share of aggregate employment. As Figure 1b shows in the early 1980s, only around one-third of firms were 11 or more years old (what we call mature firms), while by 2011 almost half of all firms were 11 or more years old. The employment share of mature firms increased from about 65 percent in the late 70s to almost 80 percent by 2011. These patterns are broad-based across sectors and geographic areas and are not due to a compositional shift in economic activity.

While these two observations are closely related, they do not necessarily imply each other. For example, the decline in firm entry could coincide with a shift towards higher quality entrants with higher survival probabilities or higher expected employment growth offsetting the declining entrant share. To isolate the margins of change, we write down a simple model of firm dynamics where employment shares by firm age are determined by the history of firm survival and employment growth by age groups in addition to the entire history of firm startup rates. Using the Census Bureau Business Dynamics Statistics (BDS) we measure firm survival and employment growth by age group and show that aside from cyclical fluctuations, both survival and growth have remained relatively constant within age group. In other words, despite the pronounced decline in the startup
rate, conditional on age, the dynamics of incumbent firms are approximately stationary. The shift in employment shares of young and mature firms over this period is almost entirely determined by the cumulative effects of the decline in the startup rate, which we refer to as the “startup deficit”.

The effect of the startup deficit—both its direct effect and its cumulative indirect effect vis-a-vis the growing share of mature firms—on aggregate employment dynamics is evident in both the trend and the cycle. We first consider the trend growth rate of employment. The trend decline in startups has an immediate negative impact on employment due to the outsized role firm entry plays in net employment creation. In addition, it has a delayed positive impact by gradually shifting the distribution of employment towards mature firms, which actually have higher growth rates than young incumbents because of a much lower exit rate. Our analysis shows that the negative immediate effect is overwhelmingly larger than the delayed positive effect thereby causing a decline in the trend growth rate of employment. To quantify the importance of the startup deficit on employment, we compute counterfactual employment paths keeping the startup rate and age distribution of firms at their 1987 levels. We show that the effect is quantitatively significant, implying a peak employment of around 6.8 percent higher in 2008 than it was in the data.

We then consider the business cycle implication of growing startup deficit. To show that aging could matter for cyclical employment dynamics, we show that the growth rate of young firms covaries more strongly with the overall economy. We do so by exploiting the aggregate time series variation on U.S. business cycles as well as cross state variation in local economic conditions. For both, we find that young businesses are more cyclically sensitive. This finding implies that the shift of employment from younger to older firms implies a decline in the cyclical sensitivity of employment.

To summarize, there are two related forces that affect employment dynamics over the business cycle. The first is the decline in employment contribution from firm entry which amplifies the response of employment to output contractions and dampens employment growth during expansions. The second is the gradual aging of firms which acts to decrease the cyclical sensitivity of employment. Put together these two forces imply a decline in trend growth rate employment and a decoupling of employment and output growth during recoveries. This decoupling, which is also referred to as jobless recoveries has been an important feature of the U.S. economy starting in 1990s.

Finally, we consider the Great Recession. This was relative to previous business cycles a significant shock. Yet despite its severity, the employment decline and recovery is inline with what we would expect given the secular decline in new businesses and slowing dynamism at incumbent firms. We compare the actual employment dynamics of the recovery from the Great Recession against two counterfactual worlds. In the first we ask what if one could increase the pace of business creation given the current distribution of incumbent firms. In the second we recognize that the distribution of incumbent firms reflects the cumulative effects of a thirty-year startup deficit. We ask what if the pace of business creation had been constant over this period. The total effect of both increasing

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1These last two requirements are consistent with the conclusions of Haltiwanger, Jarmin, and Miranda (2013) and Fort, Haltiwanger, Jarmin, and Miranda (2013), who using the same population of firms establish the higher average growth rate among young businesses as well as relatively stronger cyclical growth rates relative to older firms.
the pace of startups and reallocating the pool of businesses towards a more balanced composition of young and old firms results in an employment recovery (at least to the pre-recession peak) a full two years ahead of the current recovery.

Our paper is closely related to the emerging literature on the declining pace of reallocation in the U.S. economy. Decker, Haltiwanger, Jarmin, and Miranda (2014), Davis and Haltiwanger (2014), Haltiwanger (2013), Hathaway and Litan (2014), Lazear and Spletzer (2013) also document the decline in firm startups and emphasize that aging of firms accounts for a substantial decline in the overall pace of reallocation, i.e., gross job flows. Instead, our work considers the effect of declining startups and aging of firms on net employment growth. This allows us to provide a new perspective on jobless recoveries by linking the changes on firm dynamics to the changing cyclical behavior of employment growth. In that sense, our work is closely related to the literature on jobless recoveries and complements the structural change explanations (Groshen and Potter, 2003 and Jaimovich and Siu, 2012) and reorganization and adjustment costs-based explanations (Bachmann, 2011, Berger, 2012, and Koenders and Rogerson, 2005).

Our work also builds on the literature that considers the differential impact of business cycles on different types of firms to study the propagation and impact of business cycle shocks. See for example Moscarini and Postel-Vinay (2012), Gertler and Gilchrist (1994), Shaper (1994). While most of the earlier literature focused on firm size, our focus is on firm age similar to a recent study by Fort et al. While we believe that firm size can captures some of the differences in growth potential, credit access, or size of consumer base for firms, firm age is a better proxy as discussed by Haltiwanger, Jarmin, and Miranda (2013). Some of our findings also resonate with the small literature that analyzed the effect of aging of the work force on business cycle volatility. In particular, Gomme, Rogerson, Rupert, and Wright (2005), Clark and Summers (1981), Ríos-Rull (1996), Jaimovich and Siu (2009), Lugauer (2012) examined how the aging of the labor force acts as a stabilizing force for business cycle volatility. While we find a similar stabilizing effect through the shift of employment from younger to mature firms, we uncover an additional effect that has an opposite effect, which is the decline in firm entry.

Collectively our findings suggest that simply comparing the experiences of employment dynamics across recent business cycles may be misleading. Each business cycle in the last thirty years has shocked a different age configuration of employer firms. Even for roughly comparable business cycle shocks, it would be surprising if the outcomes were the same! Increasingly job-less recoveries are understandable when we account for the shifts in entry and its cumulative effects on the stock of incumbent firms.

2 Changing Firm Demographics

We document and analyze two striking demographic changes of US employers since the early 1980s using Census administrative data. In particular, we focus on the decline in firm entry and the shift of employment away from new and young firms towards mature firms. This change is significant for
business cycle dynamics and longer run growth as we show in the next sections. Here we document the demographic changes of US employers in detail. We present a simple analytical framework in order to decompose the margins underlying the changes in the employment shares across age groups. Interestingly, despite big changes in the age composition of employers, conditional on firm age, firm dynamics have remained relatively stationary. The growing share of old firms results almost entirely from the cumulative effects of a long run decline in the firm birth rate.

2.1 Data Description

We use data on employer businesses from the US Census Bureau Longitudinal Business Database (LBD) and its public use data product the Business Dynamics Statistics (BDS). This administrative database covers nearly every employer business in the US. The data are based on a longitudinally-linked version of the Census Bureau’s business register that includes all private-sector nonagricultural establishments with paid employees. Multiple establishments owned by the same firm are linked through their ownership information. This is an important detail, since we are interested in true firm startups rather than new locations (new establishments) of an existing firm. The data report the total employment of each firm on March 12 of each calendar year. The data are available for years 1977 through 2011. Firms founded prior to 1977 and still operating are part of the database, but their age is left censored.

Throughout, firm age is the age of the oldest establishment, measured from the year the establishment first reported positive employment. We further aggregate the firm age measure into three categories: startups (age 0), young, (ages 1 to 10) and mature (ages 11+). As Haltiwanger, Jarmin, and Miranda (2013) show, rich employment dynamics at new firms continue through about 10 years. Although our definition of young aggregates away some of this heterogeneity, our results are not sensitive to this choice. For our analysis, we use aggregations of employment and net job creation by year, our firm age groups, broad sector, and state. We provide additional details on the variable construction and sample restrictions in the data appendix B.

We focus primarily on employment rather than firm shares. The reasons are twofold: first the link between the behavior of aggregate employment and firm age is more straightforward; second employment is better measured in the administrative data than establishments and firms. The shifts in employment share are dramatic and consequential for the behavior of aggregate employment.

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2 The results included in this draft are from the Business Dynamic Statistics (BDS), which is a public use aggregation of the LBD by firm size and age. Our results from the firm level LBD are not yet approved for disclosure as of this writing. We include qualitative descriptions of the LBD results when appropriate. A description of the BDS and the data are available for download at http://www.census.gov/ces/dataproducts/bds/.

3 For a detailed description of the LBD see Jarmin and Miranda (2002).

4 In our work with the underlying microdata we use much more detailed age measures.

5 Establishments may be over- or under-measured as very small establishments hire or fire a single employee and go out of scope. We thank John Haltiwanger for pointing out the susceptibility of establishment and firm counts to measurement error for this reason.
Figure 2: Startup (Age 0) and Mature ((Age 11+) Employment Shares by Sector from 1977 (or 1987) to 2011

Note: US Census Bureau Business Dynamics Statistics. Startup employment in each sector or state as share of total sector or state employment. Series begins in 1977 because age 0 employers are not left censored.

2.2 Startup Deficits and Aging Firms

There has been a steady decline in firm entry rate and employment share of age 0 firms as we have seen in Figure 1a. One potential explanation for this decline is sectoral and/or geographic shifts in economic activity that has been taking place in the US economy since 1980s. To examine this channel, in Figure 2a we replicate the original figure in terms of employment share within seven different sectors. Almost every sector exhibits a similar decline. We see similar trends using more narrow industry classifications and within metropolitan areas using the firm level microdata. As one would expect, we observe an increase in the mature firm employment share within each sector as well. In Figure 2b we plot the employment share measure computed within broad sector for the same time period. In all years, there is considerable variation across sectors in the employment shares of mature firms. Manufacturing is the most mature sector, and construction is least. Nevertheless, within each industry, there is a pronounced upward trend. being the most mature sector, the shift of employment towards more mature firms is a common trend. The mature employment share increases at almost all sectors at roughly the same pace. Interestingly, the construction sector, which started with the lowest share of mature employment in 1977, experienced the steepest increase in mature employment.

We repeat the same exercise with states instead of sectors in Figures 3a and 3b. Again, there is considerable variation across states, but there is a striking comovement in employment shares of startups. Employment share of age 0 firms averaged around 0.4 in while it was below 0.4 for all

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6In appendix figure A.3 we plot the startup rates by sector for completion.
7In the appendix Figures A.1 and A.2 we provide the same plots using firm share rather than employment share. The trends are the same.
8We see similar trends using more narrow industry classifications using the firm level microdata.
2.3 A Decomposition Framework

The distribution of employment shares across businesses results from the cumulative effects of three forces. First is the startup employment share $e_t^s$, which we define as the age 0 share of total employment

$$e_t^s = \frac{E_t^0}{E_t}.$$  

The startup employment share is equivalent to the product of the birth rate (fraction of age 0 firms) and their average employment size relative to the overall average size. Since the average size of new firms and of overall firms are both relatively stable, the birth rate and the startup employment share in Figure 1b track each other closely. The second force is the survival rate $x_t^c$ defined as

$$x_t^c = \frac{F_t^c}{F_t^{(-1)}}.$$  

The changes in the age composition of employment reflect an aging in place rather than compositional shifts in economic activity in terms of sectors and geography. This finding suggests that shifts in sectoral and/or geographic composition of economic activity simply can not account for the aggregate patterns. To understand the relationship between the two observations, we now turn into understanding the margins that determine the age distribution of employment.

Note: US Census Bureau Business Dynamics Statistics. Employment mature (ages 11+) firms in each sector or state as share of total sector or state employment. Series begins in 1987 because firms aged 11+ are left censored from 1977 to 1986.
which is the fraction of firms for each age group cohort $c$ that survive from the previous year. We use the notation $F_c^t$ for the number of firms from cohort $c$ in period $t$ and $F_c^{t(-1)}$ for the number of firms from the same cohort in the previous year $t-1$. The third and final force is the growth in average size within each age group cohort $c$. We refer to this as the conditional growth rate $n_t$ and define it as

$$1 + n_t^c = \frac{N_t^c}{N_{t(-1)}^c},$$

where $N_t^c$ is the average employment of cohort $c$ firms in period $t$, and $N_{t(-1)}^c$ is the previous year average size holding the cohort fixed as usual. Higher order moments of the size and growth rate distribution are also important for the rich heterogeneity within cohorts, but it will be enough for our purposes to work in terms of averages.\(^9\)

Keeping track of $e_t^s$, $x_t$, and $n_t$ over time is enough to determine the age and size composition of employer businesses in each year, and these variables are all easily computed in the BDS.\(^10\)

We can write the law of motion for the distribution of employment shares across age groups as an exact decomposition. For simplicity, we use only three age groups of firms: startups (age 0) $e_t^s \equiv e_t^0$, young (ages 1 to 10) $e_t^y \equiv \sum_{a=1}^{10} e_t^a$, and mature (ages 11+) $e_t^m = \sum_{a \geq 11} e_t^a$. The mature grouping is straightforward. After 10 years much of the dynamism in a firm’s lifecycle documented in Haltiwanger, Jarmin, and Miranda (2013) subsides and the typical firm in this group looks relatively similar across ages. The young group definition is somewhat broad and aggregates much of the rich heterogeneity and dynamism among young firms into a single category. This turns out to be a reasonable simplification for our analysis, since the relative differences within the young age group have remained somewhat stable. We have repeated the decomposition exercises with more disaggregated age groups for young firms and find similar results.\(^11\)

The exact law of motion of the overall distribution of employment shares depends on the age $a$ specific unconditional growth rates $g_t^a$. For example for young firms

$$e_t^y = \sum_{a=1}^{10} e_t^{a-1} \frac{1 + g_t^a}{1 + g_t}.$$  

However, we can reformulate the law of motion entirely in terms of age group employment shares and growth rates.\(^12\) To do this we need to be careful of compositional changes across age groups since firms that were age 10 in period $t-1$ are no longer in the young firm category in period $t$. For this purpose we introduce notation $q_{t-1}^y$ to identify the fraction of age group $y$ in $t-1$ cohort

\(^9\)We address the role of heterogeneity at the firm level in ongoing work.

\(^{10}\)For details on the variable construction, please refer to the Data Appendix B.

\(^{11}\)In appendix C we also provide the decomposition by exact age group, which we use with the restricted access microdata to verify that our conclusions are unchanged.

\(^{12}\)Note that

$$1 + g_t^y = \frac{\sum_{a=1}^{10} E_t^a}{\sum_{a' \neq 1}^{10} E_{t'-1}^{a'-1}} = \sum_{a=1}^{10} \sum_{a' \neq 1}^{10} E_{t'-1}^{a'-1} \frac{E_t^a}{E_{t'-1}^{a'-1}} = \sum_{a=1}^{5} \frac{e_t^a}{e_{t-1}^a} (1 + g_t^a).$$
that remains in the $y$ age group in year $t$. Then

$$q^y_{t-1} e^y_{t-1} = \sum_{a=1}^{9} e^a_{t-1},$$

and for young firms we may write

$$e^y_t = \left(e^s_{t-1} + q^y_{t-1} e^y_{t-1}\right) \frac{1 + g^y_t}{1 + g_t}.$$

Moreover, since the gross unconditional growth rate $1 + g_t$ is the product of the survival rate and the gross conditional growth rate $1 + n_t$, we can express the growth rate of young firms in terms of firm survival $x_t$ and conditional growth $n_t$

$$e^y_t = \left(e^s_{t-1} + q^y_{t-1} e^y_{t-1}\right) \frac{x^y_t (1 + n^y_t)}{1 + g_t}. \tag{1}$$

Similarly, for the mature (ages 11+) group then

$$e^m_t = \left(1 - q^y_{t-1}\right) e^m_{t-1} + e^m_{t-1} \frac{x^m_t (1 + n^m_t)}{1 + g^m_t}. \tag{2}$$

If we define $e_t = (e^s_t, e^y_t, e^m_t)$ as the vector of employment shares across age groups we can define a non stationary transition matrix $P_t$

$$P_t = \begin{bmatrix} 0 & x^y_t (1 + n^y_t) & 0 \\ 0 & q^y_{t-1} x^y_t (1 + n^y_t) & (1 - q^y_{t-1}) x^m_t (1 + n^m_t) \\ 0 & 0 & x^m_t (1 + n^m_t) \end{bmatrix}$$

and write the law of motion for employment shares

$$e_t = \frac{1}{1 + g_t} P_t e_{t-1} + (1,0,0)' e^s_t. \tag{3}$$

Writing (3) as a $MA(\infty)$ it is clear how the current employment share distribution depends on the history of startup employment and sequences of firm survival and growth

$$e_t = \sum_{j=0}^{\infty} \left( \prod_{k=0}^{j-1} \frac{P_{t-k}}{1 + g_t - k} \right) (1,0,0)' e^s_{t-j}.$$

It will turn out that $P_t$ is close to stationary and that any fluctuations in survival and growth are second order to a trend decline in $e_t$. This decomposition framework could be equivalently formulated as the reduced form statistical model for a model of firm dynamics with entry and exit and a stochastic lifecycle component where $1 - q^y_{t-1}$ is the probability a young firm in $t-1$ becoming
a mature firm.\textsuperscript{13} The data place an important restriction on firm dynamics embedded in the age dependence of $x_t$ and $n_t$.

2.4 Margins of Adjustment

Using this framework we decompose shifts in the distribution of employment shares over time into contributions from the sequence of startup employment shares $e^{s}_t$, survival rates $x_t$ and conditional growth rate $n_t$ by age group.\textsuperscript{14} We show that despite cyclical fluctuations in the survival and growth margins encoded in $P_t$, the primary determinant of the expanding mature employment share has been the cumulative effect of the decline in startups since the 1980s.

2.4.1 Incumbent Survival and Growth

In Figure 4 we plot the one-year probability of survival $x_t$ of firms from year $t-1$ to $t$ by age group. Consistent with existing evidence on selection in Evans (1987) and Dunne, Roberts, and Samuelson (1988) for the manufacturing sector, the exit hazard for US firms declines predictably with age.\textsuperscript{15} Measured over the 1987 to 2011 period, the within age group survival probabilities are 88 percent for younger firms and 95 percent for mature firms.\textsuperscript{16} The survival rates are also mildly procyclical, showing dips in recession years. Even with this cyclicality, the within-age group survival rates are remarkably stable over this period. We confirm this stability in Table 1 where we fit a linear trend to survival rates $x_t$ by age group. Columns (1) to (3) report the estimated coefficient on the linear trend when using just annual aggregates, annual aggregates by sector, and annual aggregates by state. The estimated coefficients are almost all both statistically and quantitatively insignificant.\textsuperscript{17} The high $R^2$ confirms the stability of survival rates around their long run averages.

The specification using the national employment shares in column (1) does show a slight downward trend in young firm survival. The eye picks this up as well in Figure 4 starting in the mid-2000s. This decline appears driven by a recent shift in the survival rates of very young firms. The young age group combines the first 10 years of a firm’s life, a period with substantial heterogeneity and selection. In Appendix A we disaggregate the young age group and examine the survival rates more closely. We plot the survival rates by these age groups in Figure A.5, and in Table A.2 we estimate the same linear trends using disaggregated individual ages 1 to 5 and a medium age group of ages 6 to 10. In both the figure and the regression results, we see some evidence for a persistent decline in both early (age 1) and medium term survival. Although it is of independent interest, this recent decline has very little effect on our results.

\textsuperscript{13}This is similar to the framework used by Gertler (1999) for workers.
\textsuperscript{14}The fraction $q_{t-1}^{y}$ also adjusts with shifts in survival, growth, and startups to reflect the shifting age composition within the young age group.
\textsuperscript{15}In the appendix figure A.6 we show that the same pattern holds even within the disaggregated young age group.
\textsuperscript{16}These results are virtually identical if we exclude 2008 to 2011.
\textsuperscript{17}In the appendix table A.1 we repeat the same estimation omitting 2008 to 2011 and the results are unchanged.
Figure 4: One-year survival rates of young (ages 1 to 10) and mature (ages 11+) firms

Note: US Census Bureau Business Dynamics Statistics. Fraction of young and mature cohort that survived from previous year. Series begins in 1987 because firms aged 11+ are left censored from 1977 to 1986.

Table 1: Estimated linear trend in survival rates $x_t$ and conditional employment growth rates $n_t$ by age group

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<td>(3)</td>
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<td>State FE</td>
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</table>

Note: US Census Bureau Business Dynamics Statistics. Survival rate is fraction of young and mature cohort that survived from previous year. Conditional employment growth rate is the growth rate of cohort’s average size. Data are equally weighted across years and weighted by employment across sectors or states within years. Standard errors are clustered by year. Series begin in 1987 because firms aged 11+ are left censored from 1977 to 1986.
The relationship between firm age and conditional employment growth rate (or the within cohort growth in average size) is also stable. In Figure 5 we plot the one-year growth rate in average firm size by age group. The conditional growth rate of young firms fluctuates around its average value of 8.5 percent. Mature firms similarly fluctuate around their average conditional growth rate of 4.9 percent. Similar to above, Table 1 columns (4) to (6) report the estimated coefficient on a linear trend in $n_t$ by age group. For the US overall, within sector, and within state, the estimated trend coefficients are all statistically and quantitatively insignificant.

Mature firms have both a lower conditional growth rate and volatility roughly half of their younger counterparts. The first observation is consistent with Haltiwanger, Jarmin, and Miranda (2013) who show that conditional on survival young firms grow on average faster than old firms. In Appendix A we show the same patterns hold even within the disaggregated young age group. Figure A.6 plots the conditional growth rate by age, and Figure A.7 plots the average size. Although more volatile the disaggregated conditional growth rates show no evidence of a trend over this period. Even more remarkable is that over a thirty-year period, startups and young firms (conditional on survival) tend to have roughly the same number of employees.\textsuperscript{18}

The main takeaway is that amidst large changes in the age composition of firms, the within age group survival and growth rates are remarkably stable. Yes, they fluctuate as one would expect over the business cycle (a point we take up in detail in Section 3), but they fluctuate around steady averages with no sign of a trend. Interpreted through the decomposition framework in section 2.3, the matrix $P_t$ appears stationary and procyclical. Said differently, conditional on age, firm dynamics appear stationary. The evident stationary of $P_t$ contrasts starkly with the behavior of the startup employment share $e_s$.

As we have seen above, among the three margins that we discussed, the only one that showed a clear trend is the entry margin. Despite some fluctuations, survival and employment growth rates have been relatively stable. We next provide a formal decomposition of these three margins using the framework we developed in Section 2.3. In particular, we replace the sequence of survival rates and growth rates with their long run averages over this period and simulate the law of motion using only the realized history of the startup rates. In figure 6 we plot the simulated employment shares for each age group with constant survival and growth. They nearly perfectly replicate the actual evolution of these shares showing that the entry margin is the sole driver of the shift of employment towards older firms. The consequence of declining entry and relatively stable survival and growth is gradual aging of the distribution of incumbent firms. Each successive year bring a relatively smaller share of entrants, but they behave exactly as the cohorts that preceded them. Consequently, the increase in the mature firm employment share is almost entirely driven by the history of declining startup rates. Fluctuations in survival and growth over this period have almost no effect on the shifts in employment shares.

Most importantly, this decline in the young firm employment share and the requisite increase in

\textsuperscript{18}Average employment in mature firms increases since old firms remain old until they exit and the flow in of smaller young to old businesses diminishes over this period.
Figure 5: One-year growth rate of average employment size at young (ages 1 to 10) and mature (ages 11+) firms

Note: US Census Bureau Business Dynamics Statistics. Growth rate of average employment size of same cohort from previous year to current year. Average size in previous year also includes cohort’s firms that do not survive. Series begins in 1987 because firms aged 11+ are left censored from 1977 to 1986.

Figure 6: Mature employment share from 1987 to 2011 and its predicted path from constant survival and growth.

Note: US Census Bureau Business Dynamics Statistics. Actual is the mature employment shares from 1987 to 2011 measured in the BDS. The simulated mature employment share is simulated using actual sequence of startup employment and constant growth and survival rates in the law of motion. Series begin in 1987 because firms aged 11+ are left censored from 1977 to 1986.
the mature firm share is almost entirely driven by the history of declining startup rates. Fluctuations in survival and growth over this period have almost no effect on the shifts in employment shares. To show this, using our framework, we replace the sequence of survival rates and growth rates with their long run averages over this period and simulate the law of motion using only the realized history of the startup rates. In figure 6 we plot the simulated employment shares for each age group with constant survival and growth. They nearly perfectly replicate the actual evolution of these shares.

3 Effects on Aggregate Employment Dynamics

The startup deficit has reshaped aggregate employment dynamics through both its immediate impact on job creation and its long run cumulative effect on the employer age distribution. In this section we show how the startup deficit is slowing employment component of economic recoveries. The argument rests on two premises. First is the outsized role startups play in net employment creation. This is a point emphasized by Haltiwanger, Jarmin, and Miranda (2013), although they combine startups with other young firms. To do this we decompose the contributions to aggregate employment growth by age group. The second is the more pronounced cyclicality of young startups...

3.1 Aggregate Net Employment Growth

We define the aggregate employment growth rate in the traditional way as

\[ g_t \equiv \frac{E_t - E_{t-1}}{E_{t-1}}. \]

Next, we decompose \( g_t \) into the contributions from startups, young, and mature firms as

\[ g_t = g_t^s + \omega_{t(-1)}^y g_t^y + \omega_{t(-1)}^m g_t^m. \] (4)

The first term is the startup employment share (or growth rate contribution from startups) defined as

\[ g_t^s \equiv \frac{E_t^0}{E_{t-1}}. \]

The growth rates of young and mature firms are computed holding the year \( t \) age group cohort constant across years. For example, \( g_t^y \equiv \left( E_t^y - E_{t(-1)}^y \right) / E_{t(-1)}^y \) where \( E_{t(-1)}^y \) is the total employment in year \( t - 1 \) among the set of firms that are young in period \( t \). The employment shares use the same convention where \( \omega_{t(-1)}^y \) is the employment share in year \( t - 1 \) of the firms that are young in year \( t \), and \( \omega_{t(-1)}^m \) is the employment share at mature (age 11+ ) firms defined analogously. Since the shares refer to the cohort’s previous year’s employment the shares sum to 1. The shares \( \omega_{t(-1)} \)

\[ g_t^s \equiv \frac{E_t^0}{E_{t-1}}. \]

We use this traditional definition of employment growth rather than the symmetric version proposed by Tornqvist, Vartia, and Vartia (1985) and popularized by Davis, Haltiwanger, and Schuh (1996), because the link between the growth rate decomposition and the evolution of employment shares is more straightforward. Our results are robust to using a symmetric growth rate instead.
Figure 7: Startup growth rate contribution and growth rates of young (ages 1 to 10) and mature (ages 11+) firms

Note: US Census Bureau Business Dynamics Statistics. Startup growth rate contribution is job creation at age 0 firms as fraction of previous year’s total employment. Young (ages 1 to 10) and mature (ages 11+) growth rates are the growth rates of each cohort’s employment. Series begins in 1987 because firms aged 11+ are left censored from 1977 to 1986.

...evolve similarly to the employment shares $e$ in section C, except for the change in timing. 20

In figure 7 we plot the time series of $g_t^s$, $g_t^y$, and $g_t^m$. Several observations are evident in the time series. First is the large but shrinking growth contribution from startups plotted as the dotted line, which falls by almost 50 percent. Its marked decline is in contrast to the employment growth rates of the young and mature age groups which although volatile appear to fluctuate around a steady average. 21 The trend in the startup growth contribution imparts a trend to the aggregate time series which is attenuated somewhat by an increasing mature age group share. Second, the growth rates of young and mature age groups are on average negative. These growth rates are unconditional reflecting both employment destroyed at exiting firms and growth conditional on survival. 22 Finally, all three components not surprisingly comove strongly with the business cycle. Young firms appear to fluctuate more strongly with the business cycle, and we quantify the extent of this additional cyclicality in the next section using several sources of identification.

20 Note that

$$\omega_{t-1}^y = \frac{E_{t-1}^y}{E_{t-1}} = \frac{\frac{E_t^y}{1+g_t^y}}{\frac{E_t}{1+g_t}} = \frac{1+g_t^y}{1+g_t} e_t^y$$

so we can rewrite the growth rate decomposition in terms of contemporaneous employment shares

$$\frac{g_t}{1+g_t} = e_t^s + \frac{g_t^i}{1+g_t} e_t^y + \frac{g_t^m}{1+g_t} e_t^m .$$

21 Since the gross unconditional growth rate is the product of the survival rate and the conditional growth rate, this is consistent with the evidence in the previous section.

22 When startup employment is excluded from the young age group, the unconditional growth rate of young firms is lower (more negative) than the growth rate of mature firms because of their higher exit rate. In appendix figure A.9 we plot the growth rate of young firms inclusive of the startup contribution.
In summary, the growth rate decomposition reveals that the trend decline in startups is significant for the behavior of aggregate employment. The startup decline has an immediate impact through $g^y_t$ and, because reliably $g^y_t \neq g^m_t$, it has a delayed impact by gradually reweighting the combined growth rates of young and mature firms.

### 3.2 Cyclicality of Employment Growth

In this section we examine in more detail the relationship between the age group components of $g_t$ and the business cycle. We introduce cyclical fluctuations to the decomposition framework with a demeaned business cycle variable $Z_t$, and we estimate its covariance with the components of the aggregate growth rate. To do this we project the age group growth rates individually on $Z_t$ and estimate

\[
g^y_t = \alpha^y + \beta^y Z_t + \epsilon^y_t \tag{5}
\]
\[
g^m_t = \alpha^m + \beta^m Z_t + \epsilon^m_t \tag{6}
\]
\[
g^y_t = \bar{g}^y + \beta^y Z_t + \epsilon^y_t \tag{7}
\]
\[
g^m_t = \bar{g}^m + \beta^m Z_t + \epsilon^m_t \tag{8}
\]

where $\epsilon_t$ represents the component of $g_t$ that cannot be predicted by $Z_t$. We will say that young firms are more cyclical than mature firms if they load more heavily on the business cycle variable, i.e. when $|\beta^y| > |\beta^m|$. The estimation is straightforward, but it raises the question of what is an appropriate business cycle state variable? Here we follow Fort, Haltiwanger, Jarmin, and Miranda (2013) and use the change in the average level of unemployment as our preferred measure of $Z_t$. Moscarini and Postel-Vinay (2012) propose to use the annual average deviation of monthly unemployment from its HP filtered trend with a very high smoothing parameter as suggested by Shimer (2005). Our results are for the most part similar across both measures. We also use output based measures based on annual cumulative changes on real GDP and real personal income. In our estimation, the time period is one year. Our data on growth rates by age group are annual and some of the higher frequency variation from the business cycle or orthogonal factors is necessarily averaged away. Ideally, we would use higher-frequency time series, but employment disaggregations by firm age are unavailable.\(^{23}\)

Young firms are noticeably more cyclical than mature firms in the annual time series. Table 2 columns (1) and (2) present the estimated $\beta$ for young firms (upper panel A) and mature firms (lower panel B). Column (1) shows the raw scaled covariances from individually estimating equations (5) and (6). The second column uses data disaggregated into three firm employment size groups: less

\[^{23}\text{There is some hope that economists may one day have access to higher frequency US employment data by firm age, if the BLS databases are linked to the Census databases. The Census databases have reliable firm identifiers and longitudinal linkages, but are updated at lower frequency and with a lag.}\]
Table 2: Estimated cyclical sensitivity $\beta$ of net employment growth rates by age group using change in personal income as business cycle measure

<table>
<thead>
<tr>
<th>A. Young Firms (Ages 1 to 10)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\beta}_y$</td>
<td>1.047***</td>
<td>1.028***</td>
<td>0.710***</td>
<td>0.739***</td>
</tr>
<tr>
<td>$N$</td>
<td>25</td>
<td>75</td>
<td>1,275</td>
<td>3,804</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.283</td>
<td>0.829</td>
<td>0.664</td>
<td>0.756</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. Mature Firms (Ages 11+)</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\beta}_m$</td>
<td>0.567**</td>
<td>0.561**</td>
<td>0.440***</td>
<td>0.434***</td>
</tr>
<tr>
<td>$N$</td>
<td>25</td>
<td>75</td>
<td>1,275</td>
<td>3,825</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.189</td>
<td>0.684</td>
<td>0.728</td>
<td>0.767</td>
</tr>
</tbody>
</table>

- Size FE: - Yes - Yes - Yes
- Year FE: - - Yes Yes
- State FE: - - Yes Yes
- $p$-value $\hat{\beta}_y = \hat{\beta}_m$: 0.00345 0.00394 0.000410 0.0000477

Note: US Census Bureau Business Dynamics Statistics. Estimated projection by age group of net employment growth rate on the change in annual average unemployment. Unemployment rate averaged over retimed year of April to March to correspond to BDS March 12 employment measure. Data are equally weighted across years and weighted by employment across states and sizes within years. Standard errors are clustered by year. Series begin in 1987 because firms aged 11+ are left censored from 1977 to 1986.

than 20 employees, 20 to 499 employees, and 500 or more employees. The estimation includes fixed effects for each size group and clusters the standard errors by year. The time series estimates are nearly identical and imply that for any business cycle shock, young firm growth rates respond roughly 40 percent more than mature firms. These estimates are not specific to this time period or age grouping. The relatively high $R^2$, even without the size group fixed effects, shows us that for both age groups the majority of growth rate fluctuations are predicted by the business cycle.

The greater cyclicality of young firms is also robust to alternative sources of identification. Identifying the age group $\beta$ is a lot to ask of twenty-five annual observations spanning three business cycles (one of which is the Great Recession). As an alternative source of identification we use cross state $s$ variation in the business cycle variable $Z_{st}$ and the growth rates $g_{st}$. Here we project the age group growth rates on a constant, dummies for year $t$, state $s$, (optionally) size group $n$, and finally on a business cycle variable $Z_{st}$ and estimate

24They are similar to Fort, Haltiwanger, Jarmin, and Miranda (2013), who estimate the difference between $\beta^y$ for small firms and $\beta^m$ for large firms. In the robustness appendix we compare our estimates against Fort, Haltiwanger, Jarmin, and Miranda (2013) for alternative sample restrictions.
\[ g_{\text{snt}}^y = \alpha^y + \lambda^y_t + \mu^y_s + \gamma^y_n + \beta^y Z_{st} + \varepsilon^y_{\text{snt}} \]  
(9)

\[ g_{\text{snt}}^m = \alpha^m + \lambda^m_t + \mu^m_s + \gamma^m_n + \beta^m Z_{st} + \varepsilon^m_{\text{snt}} \]  
(10)

\[ g_{\text{snt}}^y = \bar{g}^y + \lambda^y_t + \mu^y_s + \gamma^y_n + \beta^y Z_{st} + \varepsilon^y_{\text{snt}} \]  
(11)

\[ g_{\text{snt}}^m = \bar{g}^m + \lambda^m_t + \mu^m_s + \gamma^m_n + \beta^m Z_{st} + \varepsilon^m_{\text{snt}} \]  
(12)

This identifies the parameter \( \beta \) from the within year and across state differences in state level annual changes in average state unemployment, time averaged over 1987 to 2011 and adjusting for permanent differences in growth rates across states. Columns (3) and (4) in table 2 present the separately estimated \( \beta \) for young (panel A) and mature (panel B) firms, with and without size group fixed effects. Again \( \beta^y \) is significantly above \( \beta^m \). Quantitatively, young firms load similarly on cross-state variation in \( Z_{st} \) as they do on time-series variation in \( Z_t \). Mature firms, however respond less than would have been predicted from the time-series, which amplifies the contrast in cyclicity between young and mature firms. In states with larger changes in unemployment relative to other states, we expect the differences in the growth rate of young firms to be nearly twice as large as the differences in the growth rate of mature firms.

### 3.3 Robustness

**Disaggregated Age Groups:** The cyclical volatility of employment growth also declines reliably with firm age. The results in table 2 may mask interesting dynamics by binning together firms as old as 10 years with the very young. In figure 8 we plot the \( \beta \) estimated for more finely disaggregated age groups. The vertical bars indicate 95 percent confidence intervals. Here the pattern of diminishing cyclicity with firm age is especially clear. The growth rates of very young firms in particular are strongly correlated with the business cycle. After one year the point estimates for \( \beta \) decline gradually with firm age. The estimated cyclicity from ages two through ten are statistically indistinguishable until reaching the mature ages 11+ group.\(^{25}\) The similar cyclical properties of most of the young age group lends support to our choice of aggregation groups.

**Time Variation in Cyclical Sensitivity of Employment by Firm Age:** The greater cyclicity of young firm employment than mature firm employment is also a robust finding in the data independent of time period. One might expect that as the business age distribution has tilted towards mature firms, general equilibrium effects might shift the cyclical properties within age group. Interestingly, this does not appear to be the case. To test the stability of the cyclical covariance term, we look for a first order shift over time in the sensitivity of either age group’s

\(^{25}\) An \( F \) test for the equality of \( \beta \) for ages 2,3,4,5 and 6-10 has a \( p \)-value of 0.686 when errors are clustered by year and a \( p \)-value of 0.453 when errors are clustered by state.
Figure 8: Plot of $\beta$ on state level change in average unemployment rate by disaggregated age group

Note: US Census Bureau Business Dynamics Statistics. Estimated $\beta$ on change in state level average unemployment rate using state level employment growth by age group from 1987 to 2011. Estimated with constant year and state effects across age groups.

growth rate to the business cycle indicator. The idea is to use the same within year and across state variation in $Z_{st}$ and allow the identified $\beta_t$ to depend on time through a linear time trend

$$\beta_t = \beta_0 + \beta_1 t.$$  \hfill (13)

We re-estimate equations (9) and (10) where we allow $\beta$ to include a trend component as in (13). Table 3 reports the estimated linear trend component $\beta_1$ separately estimated for young (in the first two columns) and mature (in the second two columns) firms. Columns (2) and (4) use additional variation across firm size groups and condition on firm size fixed effects. In all columns, the point estimates show a small increase in the cyclical sensitivity from 1987 to 2011, but it is statistically indistinguishable from zero.

**Role of Extensive and Intensive Margins in Differences in Cyclicality by Firm Age:**

The additional cyclicality of young firms extends to the extensive and intensive determinants of the unconditional growth rate. Our decomposition of the shifts in employment shares relied on an alternative formulation of the unconditional growth rate, namely

$$1 + g_t = x_t n_t ,$$

where we express the unconditional growth rate as the product of the cohort’s firm survival rate $x_t$, and the conditional growth rate $n_t$ which is gross growth rate of cohort’s average firm size.\(^{26}\) In

\(^{26}\)The growth in average firm size reflects both the growth rate at the cohort’s survivors and a selection effect of
Table 3: Estimated cyclical sensitivity of net employment growth rates by age group using change in personal income as business cycle measure

<table>
<thead>
<tr>
<th></th>
<th>Young Firms</th>
<th>Mature Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Linear Trend in $\hat{\beta}_t$</td>
<td>0.00288</td>
<td>-0.00264</td>
</tr>
<tr>
<td></td>
<td>(0.0113)</td>
<td>(0.0111)</td>
</tr>
<tr>
<td>$N$</td>
<td>1,275</td>
<td>3,804</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.664</td>
<td>0.756</td>
</tr>
<tr>
<td>Size FE</td>
<td>-</td>
<td>Yes</td>
</tr>
<tr>
<td>Sector FE</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>State FE</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Note: US Census Bureau Business Dynamics Statistics. Survival rate is fraction of young and mature cohort that survived from previous year. Conditional employment growth rate is the growth rate of cohort’s average size. Data are equally weighted across years and weighted by employment across sectors or states within years. Standard errors are clustered by year. Series begin in 1987 because firms aged 11+ are left censored from 1977 to 1986.

Table 4 we separately estimate versions of equations (9) and (10) where instead of the unconditional growth rates $g_t$ we use survival rates $x_t$ and conditional growth rates $n_t$ on the left hand side. Identified off of both time series $Z_t$ and cross sectional $Z_{st}$ variation, the conditional growth rates of the young firms are more cyclically sensitive than those for mature firms. The magnitudes are smaller than table 2 since the unconditional growth rates include the contributions of the survival rate, which is also procyclical. Columns (1)-(4) report the estimated $\beta$ for $x_t$. Although the evidence for procyclicality is weak in the time series, the survival rates for both young and old are notably cyclical when identified off the across state variation in $Z_{st}$. Not surprisingly, the survival rate of young firms is markedly more sensitive to the business cycle than the survival rate of mature firms. Columns (5)-(8) report the estimated $\beta$ for young and mature firms for their conditional growth rates $n_t$. The higher sensitivity of $g_t$ for young firms in table 2 is not entirely due to the survival margin. Even conditional on survival, the growth rates of young firms are more sensitive to the business cycle than those of mature firms. Nevertheless, the relative sensitivity of young survival to mature survival (anywhere from 5 to almost 15 times) is much more pronounced than for conditional growth rates (roughly 0.4 times). This is not just because young firms are more likely to exit than mature firms. Even given their higher propensity to exit, young firms are especially more likely to exit than mature firms from business cycle fluctuations.

3.4 Size versus age:

Despite large changes in the age distribution of employers in this period, the size distribution has been relatively unaffected. One might worry that as the share of older employers rises, so does the difference in average firm size between surviving and exiting firms.
the share of large firms and the differentials in cyclical sensitivity between young and mature firms reflect shifts in the size distribution. In fact, within age group the size distribution, measured roughly here by the share of small, medium, and large employers has stayed relatively constant both the in the aggregate data and at the state level. In table XXX we fit a linear time trend in the employment share of large businesses within each age group.

In summary, the declining cyclical sensitivity with firm age is a robust feature of the data. This puts important restrictions on theories of non steady-state firm dynamics. Young firms are much more sensitive to business cycle fluctuations, and disproportionately so because of the higher exit risk. Despite large compositional shifts in the economic environment, general equilibrium forces have done little to shift the extent of the age dependence in cyclicity. The result is that compositional changes due to firm aging have significant effects on the aggregate business cycle behavior of employment. The gradual rise in the mature firm share brings with it a decoupling of aggregate employment growth with the business cycle. Said differently, two recessions with identical cyclical declines in aggregate employment are in fact very far apart, and we expect the recovery employment dynamics to differ as well. In the next section we show how this aggregate decoupling from the rising mature firm share affects the dynamics of employment growth in recession and recovery periods.

4 Grown-up Employment Dynamics

The findings of the previous section suggest a change in business cycle sensitivity of aggregate employment growth due to two related factors. The first is the gradual shift of U.S. employment towards more mature firms and the second is the trend decline in the employment contribution of startups.

To fix ideas, we revisit equation 4 which decomposes growth rate of employment, $g_t$, into the contributions from startups, and existing young, and mature firms as

$$g_t = g_t^s + \omega_{t(-1)}^y g_t^y + \omega_{t(-1)}^m g_t^m \tag{14}$$

where the first term is the startup employment share and $\omega_{t(-1)}^y + \omega_{t(-1)}^m = 1$. It is useful to decompose the age-specific growth rates using equations 5 and 6:

$$g_t = g_t^s + \omega_{t(-1)}^y [g_t^y + \beta^y Z_t + \epsilon_t^y] + \omega_{t(-1)}^m [g_t^m + \beta^m Z_t + \epsilon_t^m] \tag{15}$$
Table 4: Estimated cyclical sensitivity $\beta$ of survival and conditional growth rates by age group using change in annual personal income as business cycle measure.

<table>
<thead>
<tr>
<th>Survival Rate $x_t$</th>
<th>Conditional Growth Rate $n_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>$\hat{\beta}_y$</td>
<td>$0.143$</td>
</tr>
<tr>
<td>(0.107)</td>
<td>$0.996^{**}$</td>
</tr>
<tr>
<td>$N$</td>
<td>25</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.075</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>A. Young Firms (Ages 1 to 10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\beta}_y$</td>
</tr>
<tr>
<td>(0.102)</td>
</tr>
<tr>
<td>$n_t$</td>
</tr>
<tr>
<td>$R^2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. Mature Firms (Ages 11+)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\beta}_m$</td>
</tr>
<tr>
<td>(0.0472)</td>
</tr>
<tr>
<td>$N$</td>
</tr>
<tr>
<td>$R^2$</td>
</tr>
</tbody>
</table>

Size FE - Yes - Yes - Yes - Yes
Year FE - - Yes Yes - - Yes Yes
State FE - - Yes Yes - - Yes Yes
p-value $\hat{\beta}_y = \hat{\beta}_m$ 0.142 0.0962 2.35e-07 0.00191 0.0158 0.00852 0.0841 0.00211

Note: US Census Bureau Business Dynamics Statistics. Estimated projection by age group of survival rate $x_t$ and conditional growth rate $n_t$, which are defined in text on the change in annual average unemployment. Unemployment rate averaged over retimed year of April to March to correspond to BDS March 12 employment measure. Data are equally weighted across years and weighted by employment across states and sizes within years. Standard errors are clustered by year. Series begin in 1987 because firms aged 11+ are left censored from 1977 to 1986.
and separate the trend and cyclical components

\[ g_t = g_s^t + \omega_y^{t(-1)} \bar{g}^y + \omega_m^{t(-1)} \bar{g}^m + \left( \omega_y^{t(-1)} \beta_y + \omega_m^{t(-1)} \beta_m \right) Z_t + \omega_y^{t(-1)} \varepsilon_y^t + \omega_m^{t(-1)} \varepsilon_m^t \]

The fraction of mature firms, \( \omega_m^{t(-1)} \) has been increasing since 1980s as we have seen Figure 1b. Since \( \bar{g}^m > \bar{g}^y \), this shift towards older firms implies an increase in the trend growth rate of employment. However, as we have seen in Figure ??, the startup employment growth contribution \( g_s^t \) has been trending down since early 1980s. Quantitatively, we find that the decline in the startup employment contribution is overwhelmingly larger than the increase implied by the shift towards mature firms. As a result, the trend growth rate of employment has been declining.

The cyclical terms suggest that the shift of employment towards mature firms implies a decline in cyclical sensitivity of the growth rate of employment. This follows from our finding in Table XXX that young firms’ growth rate is more cyclically sensitive than mature firms’ employment growth; \( |\beta_y| > |\beta_m| \). Since \( \omega_m^{t(-1)} \) has been increasing as a result of the aging of U.S. firms, mature firms have been becoming more important in driving the cyclical behavior of employment growth, thereby reducing the cyclical sensitivity of employment.

To summarize, there are two related forces that affect employment dynamics over the business cycle. The first is the gradual aging of firms which acts to decrease the cyclical sensitivity of employment. The second is the decline in employment contribution from firm entry which amplifies the response of employment to output contractions and dampens employment growth during expansions. Put together these two forces imply a slower employment growth during expansions for the same business cycle shock, \( Z \), while their effect is offsetting during contractions. For example, consider a recession where output declines by \( \Delta Y \). The response of existing young and mature firms’ employment to output declines has become less negative implying a smaller decline in employment for a decline of \( \Delta Y \) in real GDP (Table XXX). However, the trend decline in employment contribution of startups adds to the recessionary decline in employment, amplifying the effect of the decline in output. Since there are two offsetting effects, it is ambiguous whether the measured response of employment would be lower, higher, or little changed relative to 1980s. Now consider an expansion where output increases by \( \Delta Y \). The expansion in output corresponds to a lower increase in employment than in earlier periods in existing firms. In addition, due to the declining contribution of firm entry, employment growth is further dampened. As a result, employment growth that accompanies an expansion of \( \Delta Y \) in output becomes lower than in earlier periods.

Our analysis above suggests a decline in trend growth rate employment and a decoupling of employment and output growth during recoveries. This decoupling, which is also referred to as jobless recoveries has been an important feature of the U.S. economy starting in 1990s.
4.1 Implications for trend employment growth

In summary, the growth rate decomposition reveals that the trend decline in startups is significant for the behavior of aggregate employment. The startup decline has an immediate impact through $g^s_t$ and, because reliably $g^y_t \neq g^m_t$, it has a delayed impact by gradually reweighting the combined growth rates of young and mature firms.

The point of departure is equation XXX which decomposes employment as into startups and existing young and mature firms:

$$ E_{t+1} = S_{t+1} + E^y_{t+1} + E^m_{t+1} = S_{t+1} + (1 + g^y_t) \omega^y_{t(-1)} E_t + (1 + g^m_t) \omega^m_{t(-1)} E_t $$

Replacing conditional growth rates with the survival and employment growth rates we get

$$ E_{t+1} = S_{t+1} + x^y_{t+1} (1 + n^y_{t+1}) \omega^y_{t(-1)} E_t + x^m_{t+1} (1 + n^m_{t+1}) \omega^m_{t(-1)} E_t $$

Since we are interested in the effect of the decline in the startup rate, we decompose $S_{t+1}$ into

$$ S_{t+1} = s_{t+1} F_{t+1} N^0_{t+1} $$

where $s_{t+1}$ is the startup rate, $F_{t+1}$ is the total number of firms in the economy and $N^0_{t+1}$ is the average size of startups at time $t + 1$. This allows us to rewrite

$$ E_{t+1} = s_{t+1} F_{t+1} N^0_{t+1} + x^y_{t+1} (1 + n^y_{t+1}) \omega^y_{t(-1)} E_t + x^m_{t+1} (1 + n^m_{t+1}) \omega^m_{t(-1)} E_t $$

Our goal is to see the effect of the decline in the startup rate and the related aging of firms on employment dynamics during recoveries. To see the effect we replace the startup rate with its value in 1987 and replacing the age distribution of existing firms with the age distribution of 1987:

$$ E_{t+1} = s_{1987} F_{t+1} N^0_{t+1} + x^y_{t+1} (1 + n^y_{t+1}) \omega^y_{1987} E_t + x^m_{t+1} (1 + n^m_{t+1}) \omega^m_{1987} E_t $$

and plot the evolution of employment starting from 1987Note that in both counterfactuals, growth rates of young and mature firms, $g^y_t$ and $g^m_t$, follow the same paths they followed in the data. By doing so, we capture the business cycle fluctuations. Implicit in the assumption is the orthogonality of employment growth rates to changes in age distribution and startup rates.

Figure 9 plots the evolution of actual and counterfactual employment. Employment growth is higher under the counterfactual assumptions for the startup rate and firm age distribution. In particular, when we compare the actual level of employment at business cycle peaks of 1990, 2001, and 2008, we find that peak employment would have been, X, X, and X percent higher in 1990, 2001, and 2008 respectively.

We also examine the impact of the decline in the startup rate and aging of firms separately by changing each of them one at a time. When we only change the age distribution of firms to its 1987 composition, employment growth becomes slower as predicted by equation ??.. The effect gets
larger as the firm age distribution shifts towards older firms. When we only change the startup rate, employment growth becomes faster. The positive effect of replacing the startup rate with its 1987 level dominates the negative effect of replacing the firm age distribution with the 1987 age distribution resulting in higher employment growth in our full counterfactual which takes into account the decline in the startup rate and the consequent gradual aging of firms.

4.2 Implications for the Great Recession

Another implication of our aging is slower employment recoveries. To illustrate this point, we focus on the recovery of the 2007-09 recession and perform a similar counterfactual starting in 2007. Figure 10 shows the actual and counterfactual employment paths. Recovery takes place faster and employment reaches its pre-recession level earlier by two years. We also examine the impact of the decline in the startup rate and aging of firms separately by changing each of them one at a time. When we only change the age distribution of firms to its 1987 composition, employment growth becomes slower. When we only change the startup rate, employment growth becomes faster. The positive effect of replacing the startup rate with its 1987 level dominates the negative effect of replacing the firm age distribution with the 1987 age distribution resulting in higher employment growth in our full counterfactual which takes into account the decline in the startup rate and the consequent gradual aging of firms.

27 WE NEED TO UPDATE THIS Since the BDS data are not available yet after 2011, we use a simple method to impute the survival and employment growth rates that are consistent with the actual evolution of employment as estimated by the payroll survey. We consider the great recession as shock to the survival rate and average size for 2009 calibrated to match the size of the extensive and intensive margin adjustments we observe in the data. The survival rate shock is transient, but the average size shock reverts over 3 years as is roughly the case in the data. The 2009 to 2011 startup rate is taken from the data.
We also repeat the same exercise for the 1990-91 and 2001 recoveries using the actual survival and growth rates in the data as shown in Figures 11a and 11b. For the 1991 recession, the actual and counterfactuals employment paths are very close while the gap opens up in the 2001 recession. Employment reaches its pre-recession level half a year earlier for the recoveries of 1990-91 and 2001 recessions. When we only change the startup rate, recoveries become faster for all recessionary episodes. For all three counterfactuals, the positive effect of replacing the startup rate with its 1987 level dominates the negative effect of replacing the firm age distribution with the 1987 age distribution.

5 Reasons for the decline in the startup rate

5.1 Changing demographic composition of the U.S. population

5.2 Rising concentration of income in the entrepreneurial sector

6 Conclusions

We document an extremely broad based decline in business formation. Given that conditional on starting, firm dynamics have remained relatively constant over this thirty-year period, the cumulative effects of this decline, or startup deficit, have radically shifted the age composition of employer businesses. We describe the mechanics of how the decreased cyclicity of older businesses along with the diminishing employment flows from entering businesses have lengthened the employment component of economic recoveries over the last four business cycles. This is a natural counterpart to work by Jaimovich and Siu (2012) that make a similar observation based on occupational
(a) Actual and counterfactual evolution of employment during the recovery and expansions after the 1990-91 recession

(b) Actual and counterfactual evolution of employment during the recovery and expansions after the 2001 recession

Figure 11

Note:
Both Firms and Workers are Aging

A natural question, especially considering the robustness of the startup deficit is why has the startup rate declined so much over this period? Although surely not the entire explanation, we draw attention to two aggregate phenomenon consistent with this broad-based decline. First is low frequency demographic shifts over this period, that may have depleted the pool of potential entrepreneurs and lower wage workers for new firms. We plot in Figure 12 the startup rate against the rising age of the workforce. The second and related trend is the rising real wage of potential business founders. An implication of Lucas’s original span of control model in Lucas Jr (1978) is the sensitivity of the selection into entrepreneurship to the wage compensation as an employee. As productivity gains have raised the real wage, they may have also raised the threshold for starting a profitable business. This of course puts restrictions on the path of marginal businesses over time, which can be tested. This is an active area of research for us and the subject of a new paper.

Figure 12: Startup rate and average workforce age 1978 to 2011
References


Table A.1: Estimated linear trend in survival rates $x_t$ and conditional employment growth rates $n_t$ by age group over alternative sample years

<table>
<thead>
<tr>
<th>Linear Trend</th>
<th>Survival Rate $x_t$</th>
<th>Conditional Employment Growth Rate $n_t$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
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<tr>
<td>Young</td>
<td>3.74e-05</td>
<td>0.000103</td>
</tr>
<tr>
<td></td>
<td>(0.000169)</td>
<td>(0.000160)</td>
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<tr>
<td>Mature</td>
<td>0.000384***</td>
<td>0.000244*</td>
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<tr>
<td></td>
<td>(0.000135)</td>
<td>(0.000129)</td>
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<tr>
<td>$N$</td>
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<tr>
<td>$R^2$</td>
<td>0.988</td>
<td>0.969</td>
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<tr>
<td>Age Group FE</td>
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<td>Yes</td>
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<tr>
<td>Sector FE</td>
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<td>Yes</td>
</tr>
<tr>
<td>State FE</td>
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</table>

Note: US Census Bureau Business Dynamics Statistics. Survival rate is fraction of young and mature cohort that survived from previous year. Conditional employment growth rate is the growth rate of cohort’s average size. Data are equally weighted across years and weighted by employment across sectors or states within years. Standard errors are clustered by year. Series begin in 1987 because firms aged 11+ are left censored from 1977 to 1986.

A  Additional Tables and Figures
Table A.2: Estimated linear trend in survival rates $x_t$ and conditional employment growth rates $n_t$
by detailed age group

<table>
<thead>
<tr>
<th>Linear Trend</th>
<th>Survival Rate $x_t$</th>
<th>Conditional Employment Growth Rate $n_t$</th>
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<tr>
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<tr>
<td>Age 1</td>
<td>-0.00161***</td>
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<tr>
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<td>(0.000480)</td>
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<td>Age 2</td>
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<tr>
<td></td>
<td>(0.000292)</td>
<td>(0.000269)</td>
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<tr>
<td>Age 3</td>
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<tr>
<td></td>
<td>(0.000239)</td>
<td>(0.000214)</td>
</tr>
<tr>
<td>Age 4</td>
<td>-0.000293</td>
<td>-0.000236</td>
</tr>
<tr>
<td></td>
<td>(0.000225)</td>
<td>(0.000200)</td>
</tr>
<tr>
<td>Age 5</td>
<td>-0.000306</td>
<td>-0.000273</td>
</tr>
<tr>
<td></td>
<td>(0.000206)</td>
<td>(0.000180)</td>
</tr>
<tr>
<td>Ages 6 to 10</td>
<td>-0.000346**</td>
<td>-0.000378***</td>
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<td></td>
<td>(0.000144)</td>
<td>(0.000122)</td>
</tr>
</tbody>
</table>

| N            | 150                | 1,350            | 7,650             | 150              | 1,350            | 7,650            |
| R²           | 0.947              | 0.934            | 0.912             | 0.735            | 0.571            | 0.398            |
| Age Group FE | Yes                | Yes              | Yes               | Yes              | Yes              | Yes              |
| Sector FE    | -                  | Yes              | -                 | -                | Yes              | -                |
| State FE     | -                  | -                | Yes               | -                | -                | Yes              |

Note: US Census Bureau Business Dynamics Statistics. Survival rate is fraction of young and mature cohort that survived from previous year. Conditional employment growth rate is the growth rate of cohort’s average size. Data are equally weighted across years and weighted by employment across sectors or states within years. Standard errors are clustered by year. Series begin in 1987 because firms aged 11+ are left censored from 1977 to 1986.
Table A.3: Estimated cyclical sensitivity $\beta$ of net employment growth rates by age group using alternative output and employment based business cycle variables

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<td></td>
<td>Gross</td>
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<td>National/State</td>
<td>Differences in</td>
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<td></td>
<td></td>
<td>Income</td>
<td>Product</td>
<td>Unemployment</td>
</tr>
<tr>
<td>$\hat{\beta}_y$</td>
<td>1.047***</td>
<td>1.317***</td>
<td>-2.039***</td>
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<tr>
<td></td>
<td>(0.347)</td>
<td>(0.236)</td>
<td>(0.575)</td>
<td>(0.333)</td>
</tr>
<tr>
<td>$\hat{\beta}_m$</td>
<td>0.567**</td>
<td>0.823***</td>
<td>-1.445***</td>
<td>-0.370</td>
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<td></td>
<td>(0.233)</td>
<td>(0.153)</td>
<td>(0.425)</td>
<td>(0.227)</td>
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<tr>
<td>$p$-value for $\hat{\beta}_y = \hat{\beta}_m$</td>
<td>0.00345</td>
<td>0.000528</td>
<td>0.0186</td>
<td>0.918</td>
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A. National Measures

B. State Level Measures

<table>
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<tr>
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</thead>
<tbody>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{\beta}_y$</td>
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<td>0.426***</td>
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<td>-0.943***</td>
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<tr>
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<td>(0.0969)</td>
<td>(0.0670)</td>
<td>(0.354)</td>
<td>(0.279)</td>
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<tr>
<td>$\hat{\beta}_m$</td>
<td>0.440***</td>
<td>0.263***</td>
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<td>-0.685***</td>
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<tr>
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<td>(0.0428)</td>
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<td>$p$-value for $\hat{\beta}_y = \hat{\beta}_m$</td>
<td>0.00041</td>
<td>0.000438</td>
<td>0.000426</td>
<td>0.0767</td>
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Note: US Census BDS, Bureau of Economic Analysis, Bureau of Labor Statistics. Estimated projection by age group of net employment growth rate on the indicated business cycle measures. Unemployment rate and HP filtered unemployment averaged- and personal income and gross domestic product summed- over retimed year of April to March to correspond to BDS March 12 employment measure. Gross state product is measured over previous calendar year. Data are equally weighted across years and weighted by employment across states and sizes within years. Standard errors are clustered by year. Series begin in 1987 because firms aged 11+ are left censored from 1977 to 1986.
Table A.4: Estimated cyclical sensitivity $\beta$ of detrended startup employment share

<table>
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<th>(4)</th>
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</thead>
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<tr>
<td>$\hat{\beta}^s$</td>
<td>0.00721</td>
<td>0.00857</td>
<td>0.0371***</td>
<td>0.0266***</td>
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<tr>
<td></td>
<td>(0.0475)</td>
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<td>(0.0112)</td>
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<td>$N$</td>
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<td>31</td>
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<td>$R^2$</td>
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<td>0.000</td>
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<td>0.536</td>
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<td>Year FE</td>
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<td>Detrending</td>
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<td>HP</td>
<td>Linear</td>
<td>HP</td>
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</table>

Note: US Census BDS, Bureau of Economic Analysis, Bureau of Labor Statistics. Estimated projection of detrended startup employment share on national and state personal income, which are measured over retimed year of April to March to correspond to BDS March 12 employment measure. HP filtered startup employment shares computed with $\lambda = 100$. Data are equally weighted across years and weighted by employment across states and sizes within years. Standard errors are clustered by year. Series begin in 1987 because firms aged 11+ are left censored from 1977 to 1986.

Figure A.1: Mature firm share within each sector

Note: US Census Bureau Business Dynamics Statistics. Mature firm share is number of sector’s mature (age 11+) firms as fraction of total sector firms in each year.
Figure A.2: Mature firm share within each state

Note: US Census Bureau Business Dynamics Statistics. Mature firm share is number of state’s mature (age 11+) firms as fraction of total state firms in each year.

Figure A.3: Startup rate within each sector

Note: US Census Bureau Business Dynamics Statistics. Startup rate is number of sector’s startup (age 0) firms as fraction of total sector firms in each year.
Figure A.4: Startup rate within each state

Note: US Census Bureau Business Dynamics Statistics. Startup rate is number of state’s startup (age 0) firms as fraction of total state firms in each year.

Figure A.5: One-year cohort survival rates of detailed ages 1 to 5, middle age group (ages 6 to 10) and mature age group (ages 11+) firms

Note: US Census Bureau Business Dynamics Statistics. Fraction of each cohort’s firms that survived from previous year. The middle (ages 6 to 10) and mature (ages 11+) groups are left censored from 1977 to 1986.
Figure A.6: One-year cohort growth rate of average employment of detailed ages 1 to 5, middle age group (ages 6 to 10) and mature age group (ages 11+) firms

Note: US Census Bureau Business Dynamics Statistics. Growth rate of average employment size of same cohort from previous year to current year. Average size in previous year also includes cohort’s firms that do not survive. The middle (ages 6 to 10) and mature (ages 11+) groups are left censored from 1977 to 1986.

Figure A.7: Average employment by detailed age groups

Note: US Census Bureau Business Dynamics Statistics.
Figure A.8: Actual and predicted startup, young, and mature employment shares

Note: US Census Bureau Business Dynamics Statistics. Startup employment share from 1977 to 2011 and young and mature employment shares from 1987 to 2011 are actual data and measure in the BDS. The model-based employment shares are predicted forwards and backwards from 1987 employment share distribution using actual sequence of startup shares and constant growth and survival rates in the law of motion. Young and mature series begin in 1987 because firms aged 11+ are left censored from 1977 to 1986.

Figure A.9: Growth rates of combined young and new firms (ages 0 to 10) and mature (ages 11+) firms

Note: US Census Bureau Business Dynamics Statistics. Growth rate of young and new firms is total employment at young and new firms relative to the total employment of the young cohort in the previous year. Series begins in 1987 because firms aged 11+ are left censored from 1977 to 1986.
B Data Appendix

We measure the following in the data

\( g_{yt}^{m}, g_{t}^{m} \) national growth rate of age group \( y,m \) firms from \( t \) to \( t - 1 \)

\( g_{yt}^{s}, g_{t}^{s} \) state level growth rate of age group \( y,m \) firms from \( t \) to \( t - 1 \)

\( Z_{t} \) national business cycle indicator (see below) in period \( t \)

\( Z_{ts} \) state level business cycle indicator in period \( t \)

\( x_{yt}^{m}, x_{t}^{m} \) national fraction of surviving age group \( y,m \) firms from \( t \) to \( t - 1 \)

\( N_{t}^{s}, N_{t}^{y}, N_{t}^{m} \) average firm size of age group \( s,y,m \) firms in \( t \)

\( N_{t}^{y(-1)}, N_{t}^{m(-1)} \) average firm size of same cohort of firms in \( t - 1 \)

\( e_{yt}^{s}, e_{t}^{s}, e_{t}^{m} \) national employment share of age group \( s,y,m \) firms in \( t \)

\( e_{t}^{y(-1)}, e_{t}^{m(-1)} \) national employment share of time \( t \) group \( y,m \) firms in \( t - 1 \)

\( e_{ts}^{s}, e_{ts}^{y}, e_{ts}^{m} \) state employment share of age group \( s,y,m \) firms in \( t \)

\( \omega_{t}^{y(-1)s}, \omega_{t}^{m(-1)s} \) state employment share of time \( t \) group \( y,m \) firms in \( t - 1 \)

\( f_{t}^{s}, f_{t}^{y}, f_{t}^{m} \) national firm share of age group \( s,y,m \) firms in \( t \)

B.1 Business Dynamics Statistics

Describe procedure for BDS

B.2 Longitudinal Business Database

Describe procedure for LBD

C Full Decomposition of Employment Distribution

C.1 Exact Ages 1 to 10 and 11+

Let \( E_{t}^{a} \) represent total employment of age \( a \) firms in year \( t \). Define the age \( a \) employment share \( e_{t}^{a} = E_{t}^{a} / E_{t} \), and let \( e_{t} \) be a vector of employment shares by age

\[
 e_{t} \equiv \begin{pmatrix}
 e_{t}^{0} \\
 e_{t}^{1} \\
 \vdots \\
 e_{t}^{10} \\
 e_{t}^{m} 
\end{pmatrix},
\]
where $e^{11+}_t$ is the employment share of mature (11+) firms. We take the startup employment share $e^0_t$ as exogenous. The law of motion for the employment share $e^a_t$ is

$$e^a_t = \frac{E^{a-1}_t}{E_{t-1}} (1 + g^a_t) \frac{E_{t-1}}{E_t} = e^{a-1}_{t-1} \frac{1 + g^a_t}{1 + g_t} \quad 1 \leq a < 11,$$

(16)

where $g^a_t = E^a_t / E^{a-1}_{t-1}$ is the age $a$ employment growth rate and $g_t = E_t / E_{t-1}$. For the mature firms where $a \geq 11$ the law of motion is

$$e^{11+}_t = \left(e^{10}_{t-1} + e^{11+}_{t-1}\right) \frac{1 + g^m_t}{1 + g_t},$$

(17)

where $g^{11+}_{t-1}$ is the growth rate of firms that were 11+ in year $t-1$.

We reformulate the law of motion in terms of firm dynamics by defining the number of age $a$ firms as $F^a_t$, and the average employment of age $a$ firms $N^a_t$. Then

$$1 + g^a_t = \frac{E^a_t}{E^{a-1}_{t-1}} = \frac{E^a_t N^a_t}{E^{a-1}_{t-1} N^{a-1}_{t-1}} = x^a_t \frac{N^a_t}{N^{a-1}_{t-1}},$$

where $x^a_t$ is the survival rate of firm from age $a-1$ to age $a$. For mature firms

$$1 + g^{11+}_t = x^{11+}_t \frac{N^{11+}_t}{N^{10+}_{t-1}},$$

where importantly $x^{10+}_t$ is the survival rate for the cohort of firms 11+ in year $t$, and $N^{10+}_{t-1}$ is the average employment for the year $t$ mature cohort in $t-1$, i.e., when aged 10+ in year $t-1$.

Finally, we define a transition matrix $P_t$ in terms of firm survival and the growth in average size

$$P_t = \begin{bmatrix}
0 & x^1_t & 0 & 0 & \cdots & 0 \\
0 & 0 & x^2_t & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & x^{11+}_t & N^{11+}_{t_{10+}} & \cdots \\
0 & 0 & 0 & 0 & \cdots & x^{11+}_t & N^{11+}_{t_{10+}}
\end{bmatrix}$$

and write the law of motion for the distribution of employment shares $e_t$ as

$$e_t = \frac{1}{1 + g_t} P_t e_{t-1} + (1, 0, 0, \ldots)^t e^0_t$$

(18)

where $e^0_t$ is the startup employment share.